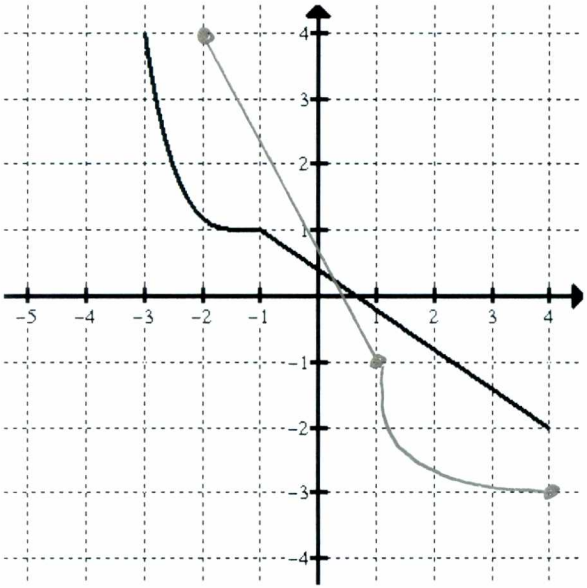
The graph of a function f is shown below. Sketch a graph of f^{-1} on the same axes.

SCORE: ____ / 14 PTS



POINTS ON f

$(-3, 4)$

$(-1, 1)$

$(4, -2)$

POINTS ON f^{-1}

$(4, -3)$

$(1, -1)$

$(-2, 4)$

Let $f(x) = \dfrac{x}{1-4x}$ and $g(x) = \dfrac{2}{x-3}$.

SCORE: ____ / 18 PTS

[a] Find $(f \circ g)(x)$. Simplify your final answer.

$$f(g(x)) = \frac{\frac{2}{x-3}}{1-\frac{8}{x-3}} \cdot \frac{x-3}{x-3} = \frac{2}{x-3-8} = \frac{2}{x-11}$$

[b] Find the domain of $f \circ g$.

DOMAIN OF $g = \{x \neq 3\}$

DOMAIN OF $f = \{x \neq \frac{1}{4}\}$

$g(x) \neq \frac{1}{4}$

$\frac{2}{x-3} \neq \frac{1}{4}$

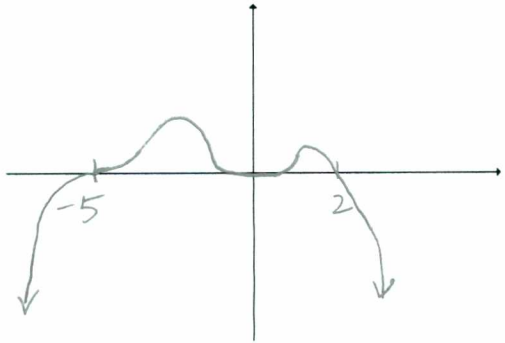
$8 \neq x-3 \longrightarrow x \neq 11$

DOMAIN OF fog

$= \{x \neq 3$ AND $x \neq 11\}$

Sketch the graph of $f(x) = -x^4(x-2)(x+5)^3$.

SCORE: _____ / 14 PTS



Find the remainder when $f(x) = 7x^{38} + 11x^{29} + 2x - 3$ is divided by $x + 1$.

SCORE: _____ / 10 PTS

HINT: There is a short way to solve this, and a L O N G way to solve this.

$= f(-1) = 7(1) + 11(-1) + 2(-1) - 3$

$= 7 - 11 - 2 - 3$

$= -9$

Let $f(x) = \dfrac{3-2x}{4x+5}$.

SCORE: ____ / 16 PTS

[a] Find $f^{-1}(x)$.

$$y = \frac{3-2x}{4x+5}$$

$$x = \frac{3-2y}{4y+5}$$

$$4xy + 5x = 3 - 2y$$

$$4xy + 2y = 3 - 5x$$

$$y(4x+2) = 3 - 5x$$

$$y = \frac{3-5x}{4x+2}$$

$$f^{-1}(x) = \frac{3-5x}{4x+2}$$

[b] Find the range of $f(x)$. **HINT: Why is this part [b] ?**

DOMAIN OF f^{-1}

$= \{x \neq -\frac{1}{2}\}$

RANGE OF $f = \{y \neq -\frac{1}{2}\}$

Find a polynomial with real coefficients with roots -1 and $2-3i$. Simplify your answer completely. **SCORE: ____ / 14 PTS**

$$(x-(-1))(x-(2-3i))(x-(2+3i))$$
$$=(x+1)((x-2)+3i)((x-2)-3i)$$
$$=(x+1)((x-2)^2-9i^2)$$
$$=(x+1)(x^2-4x+4+9)$$
$$=(x+1)(x^2-4x+13)$$
$$=x^3-4x^2+13x+x^2-4x+13=x^3-3x^2+9x+13$$

Find two functions f and g such that $(f \circ g)(x) = \sqrt{1+(3x^5+x)^2}$.

SCORE: _____ / 7 PTS

$f(x) = \sqrt{1+x^2}$ $\quad$ $g(x) = 3x^5+x$

Consider the polynomial $f(x) = 3x^4 - 11x^3 - 10x^2 - 47x - 15$.

SCORE: ____ / 33 PTS

[a] According to Descartes' Rule of Signs, how many possible positive and negative real roots does f have?

1 POSITIVE ROOT

3 OR 1 NEGATIVE ROOTS

$f(-x) = 3x^4 + 11x^3 - 10x^2 + 47x - 15$

[b] List all the possible rational roots of f.

$\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{3}, \pm \frac{5}{3}$

[c] Find $f(-1)$ using synthetic division. **YOU MUST USE SYNTHETIC DIVISION TO EARN ANY POINTS.**

-1	3	-11	-10	-47	-15
		-3	14	-4	51
	3	-14	4	-51	36

$f(-1) = 36$

[d] Find all roots of f.

↑ ALTERNATING SIGNS $\Rightarrow$ ALL ROOTS > -1

$-\frac{1}{3}$	3	-11	-10	-47	-15
		-1	4	2	15
	3	-12	-6	-45	0

$3x^4 - 11x^3 - 10x^2 - 47x - 15$

$= (x + \frac{1}{3})(3x^3 - 12x^2 - 6x - 45)$

$= (3x+1)(x^3 - 4x^2 - 2x - 15)$

ONLY NEED TO TRY $x = 1, 3, 5, 15$

5	1	-4	-2	-15
		5	5	15
	1	1	3	0

$= (3x+1)(x-5)(x^2 + x + 3)$

$x^2 + x + 3 = 0$

$x = \frac{-1 \pm \sqrt{-11}}{2}$

$x = \frac{-1 \pm \sqrt{11}\,i}{2}$

ROOTS $= -\frac{1}{3}, 5, -\frac{1}{2} \pm \frac{\sqrt{11}}{2}i$

A corral is to be made in the shape of a rectangle with a divider running down the middle. The corral and divider together are to be made using exactly 16 meters of fencing. What should be the dimensions of the corral in order for the total area to be a maximum ?

SCORE: _____ / 14 PTS

$3x+2y=16$

$y=\frac{16-3x}{2}=-\frac{3}{2}x+8$

$A=xy=x\left(-\frac{3}{2}x+8\right)=-\frac{3}{2}x^2+8x$

VERTEX $x=-\frac{8}{-3}=\frac{8}{3}$

$y=-\frac{3}{2}\cdot\frac{8}{3}+8=4$

$\frac{8}{3}m \times 4m$