

Consider the polynomial $f(x) = 3x^4 - 11x^3 - 10x^2 - 47x - 15$.

SCORE: ____ / 33 PTS

[a] According to Descartes' Rule of Signs, how many possible positive and negative real roots does f have?

1 POSITIVE ROOT $f(-x) = 3x^4 + 11x^3 - 10x^2 + 47x - 15$
 3 or 1 NEGATIVE ROOTS

[b] List all the possible rational roots of f .

$\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{3}, \pm \frac{5}{3}$

[c] Find $f(-1)$ using synthetic division. YOU MUST USE SYNTHETIC DIVISION TO EARN ANY POINTS.

$-1 \mid 3 \quad -11 \quad -10 \quad -47 \quad -15$
 $\quad -3 \quad 14 \quad -4 \quad 51$
 $\hline 3 \quad -14 \quad 4 \quad -51 \quad \underline{36}$ $f(-1) = 36$

[d] Find all roots of f .

$\uparrow$
 ALTERNATING SIGNS $\Rightarrow$ ALL ROOTS > -1

$-\frac{1}{3} \mid 3 \quad -11 \quad -10 \quad -47 \quad -15$
 $\quad -1 \quad 4 \quad 2 \quad 15$
 $\hline 3 \quad -12 \quad -6 \quad -45 \quad \underline{0}$

$3x^4 - 11x^3 - 10x^2 - 47x - 15$
 $= (x + \frac{1}{3})(3x^3 - 12x^2 - 6x - 45)$
 $= (3x + 1)(x^3 - 4x^2 - 2x - 15)$

ONLY NEED TO
 TRY $x = 1, 3, 5, 15$

$5 \mid 1 \quad -4 \quad -2 \quad -15$
 $\quad 5 \quad 5 \quad 15$
 $\hline 1 \quad 1 \quad 3 \quad \underline{0}$

$= (3x + 1)(x - 5)(x^2 + x + 3)$

$$x^2 + x + 3 = 0$$

$$x = \frac{-1 \pm \sqrt{-11}}{2}$$

$$x = \frac{-1 \pm \sqrt{11}i}{2}$$

$$\text{ROOTS} = -\frac{1}{3}, 5, -\frac{1}{2} \pm \frac{\sqrt{11}}{2}i$$

Find two functions f and g such that $(f \circ g)(x) = \sqrt{1 + (3x^5 + x)^2}$.

SCORE: ____ / **7 PTS**

$$f(x) = \sqrt{1 + x^2}$$

$$g(x) = 3x^5 + x$$

Let $f(x) = \frac{3-2x}{4x+5}$.

SCORE: ____ / 16 PTS

[a] Find $f^{-1}(x)$.

$$y = \frac{3-2x}{4x+5}$$

$$x = \frac{3-2y}{4y+5}$$

$$4xy + 5x = 3 - 2y$$

$$4xy + 2y = 3 - 5x$$

$$y(4x+2) = 3 - 5x$$

$$y = \frac{3-5x}{4x+2}$$

$$f^{-1}(x) = \frac{3-5x}{4x+2}$$

[b] Find the range of $f(x)$. HINT: Why is this part [b]?

$$\begin{aligned} &\text{DOMAIN OF } f^{-1} \\ &= \{x \neq -\frac{1}{2}\} \end{aligned}$$

$$\text{RANGE OF } f = \{y \neq -\frac{1}{2}\}$$

Find a polynomial with real coefficients with roots -1 and $2 - 3i$. Simplify your answer completely.

SCORE: ____ / 14 PTS

$$\begin{aligned} & (x - (-1))(x - (2 - 3i))(x - (2 + 3i)) \\ &= (x + 1)((x - 2) + 3i)((x - 2) - 3i) \\ &= (x + 1)((x - 2)^2 - 9i^2) \\ &= (x + 1)(x^2 - 4x + 4 + 9) \\ &= (x + 1)(x^2 - 4x + 13) \\ &= x^3 - 4x^2 + 13x + x^2 - 4x + 13 = x^3 - 3x^2 + 9x + 13 \end{aligned}$$

Find the remainder when $f(x) = 7x^{38} + 11x^{29} + 2x - 3$ is divided by $x + 1$.

SCORE: ____ / 10 PTS

HINT: There is a short way to solve this, and a L O N G way to solve this.

$$\begin{aligned} &= f(-1) = 7(1) + 11(-1) + 2(-1) - 3 \\ &= 7 - 11 - 2 - 3 \\ &= -9 \end{aligned}$$

Let $f(x) = \frac{x}{1-4x}$ and $g(x) = \frac{2}{x-3}$.

SCORE: ____ / 18 PTS

[a] Find $(f \circ g)(x)$. Simplify your final answer.

$$f(g(x)) = \frac{\frac{2}{x-3}}{1 - \frac{8}{x-3}} \cdot \frac{x-3}{x-3} = \frac{2}{x-3-8} = \frac{2}{x-11}$$

[b] Find the domain of $f \circ g$.

$$\text{DOMAIN OF } g = \{x \neq 3\}$$

$$\text{DOMAIN OF } f = \{x \neq \frac{1}{4}\}$$

$$g(x) \neq \frac{1}{4}$$

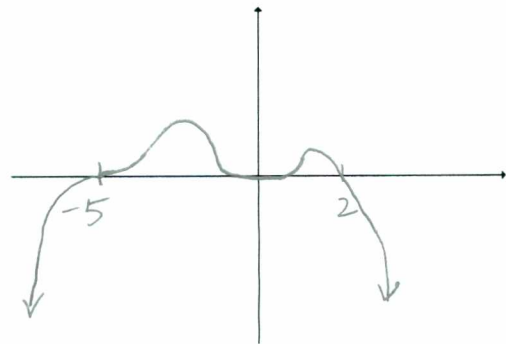
$$\frac{2}{x-3} \neq \frac{1}{4}$$

$$8 \neq x-3 \longrightarrow x \neq 11$$

$$\begin{aligned} \text{DOMAIN OF } f \circ g \\ = \{x \neq 3 \text{ AND } x \neq 11\} \end{aligned}$$

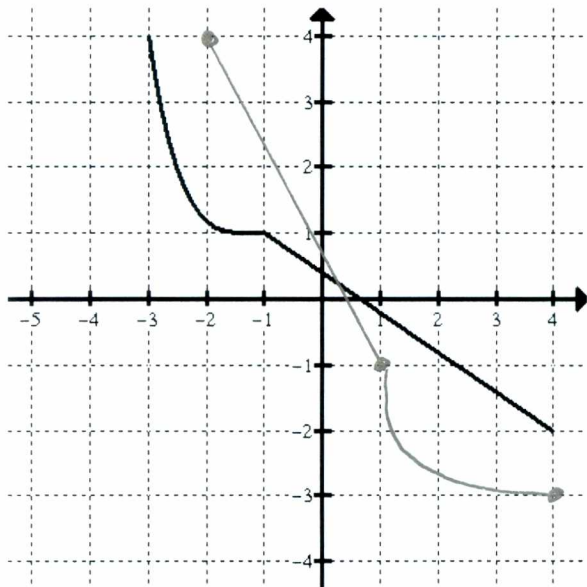
Sketch the graph of $f(x) = -x^4(x-2)(x+5)^3$.

SCORE: ____ / 14 PTS



The graph of a function f is shown below. Sketch a graph of f^{-1} on the same axes.

SCORE: ____ / 14 PTS



POINTS
ON f
 $(-3, 4)$
 $(-1, 1)$
 $(4, -2)$

POINTS
ON f^{-1}
 $(4, -3)$
 $(1, -1)$
 $(-2, 4)$

A corral is to be made in the shape of a rectangle with a divider running down the middle. The corral and divider together are to be made using exactly 16 meters of fencing. What should be the dimensions of the corral in order for the total area to be a maximum ?

SCORE: ____ / 14 PTS

$$3x + 2y = 16$$

$$y = \frac{16 - 3x}{2} = -\frac{3}{2}x + 8$$

$$A = xy = x\left(-\frac{3}{2}x + 8\right) = -\frac{3}{2}x^2 + 8x$$

$$\text{VERTEX } x = -\frac{8}{-\frac{3}{2}} = \frac{8}{3}$$

$$y = -\frac{3}{2} \cdot \left(\frac{8}{3}\right)^2 + 8 = 4$$

$$\frac{8}{3} \text{ m} \times 4 \text{ m}$$

