Consider the function $f(x)=\dfrac{8-2x^2}{x^2+x-2}$. $=\dfrac{-2(x+2)(x-2)}{(x+2)(x-1)}=\dfrac{-2(x-2)}{x-1}$

SIMPLIFIED

SCORE: _____ / 21 PTS

[a] Find the domain of f.

$\{x \neq -2 \text{ AND } x \neq 1\}$

[b] Find all intercepts of f.

X-INT: $x=2$ $(2,0)$

y-INT: $f(0)=\frac{8}{-2}=-4$ $(0,-4)$

[c] Find all asymptotes (vertical, horizontal and/or slant) of f.

V.A $x=1$

H.A. $y=\frac{-2}{1}=-2$

NO S.A.

Write $\log_2 \dfrac{32x}{\sqrt{y^3(x^5+1)}}$ in terms of the simplest possible logarithms.

SCORE: _____ / 8 PTS

$\log_2 32+\log_2 x-\frac{3}{2}\log_2 y-\frac{1}{2}\log_2(x^5+1)$

$=5-\log_2 x-\frac{3}{2}\log_2 y-\frac{1}{2}\log_2(x^5+1)$

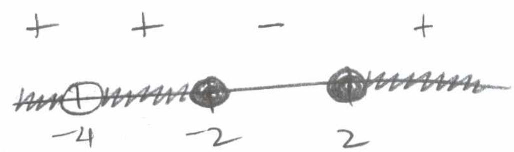
Write $\frac{1}{3}[2\ln x + 5\ln(x^2+1)] - 3\ln(2x+7)$ as the logarithm of a single quantity. **SCORE: _____ / 6 PTS**

$\ln \dfrac{x^{\frac{2}{3}}(x^2+1)^{\frac{5}{3}}}{(2x+7)^3}$ OR $\ln \dfrac{\sqrt[3]{x^2(x^2+1)^5}}{(2x+7)^3}$

Find the domain of $f(x) = \sqrt{\dfrac{x^2-4}{x^2+8x+16}}$. **SCORE: _____ / 21 PTS**

$\dfrac{x^2-4}{x^2+8x+16} \geq 0$

$\dfrac{(x+2)(x-2)}{(x+4)^2} \geq 0$



$(-\infty,-4) \cup (-4,-2] \cup [2,\infty)$

Solve $\log_2(1-x) + \log_2(5-x) = 5$. **SCORE: _____ / 21 PTS**

$\log_2 (1-x)(5-x) = 5$

$(1-x)(5-x) = 2^5$

$5 - 6x + x^2 = 32$

$x^2 - 6x - 27 = 0$

$(x-9)(x+3) = 0$

$x = 9$ or $\boxed{x = -3}$

CHECK:

$\cancel{x = 9 \quad \log_2(-8)}$

$x = -3 \quad \log_2 4 + \log_2 8$

$= 2 + 3$

$= 5$

Find all asymptotes (vertical, horizontal and/or slant) of $f(x) = \dfrac{7+13x-8x^3}{2x^2-x-3}$. **SCORE: _____ / 21 PTS**

V.A. $2x^2-x-3=0$

$$x = \frac{1 \pm \sqrt{1-4(2)(-3)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{25}}{4}$$

$$= \frac{1 \pm 5}{4} = \frac{3}{2} \text{ OR } -1$$

$x = \frac{3}{2}$

$x = -1$

NO H.A.

S.A.

$$\begin{array}{r} -4x-2 \\ 2x^2-x-3 \overline{)\,-8x^3+0x^2+13x+7} \\ \underline{-8x^3+4x^2+12x} \\ -4x^2+x+7 \\ \underline{-4x^2+2x+6} \\ -x+1 \end{array}$$

$y = -4x-2$

Animals of an endangered species were released into a game preserve, and the population t months later fit the logistic model $p(t) = \dfrac{1320}{1+10e^{-0.09t}}$.

SCORE: _____ / 21 PTS

[a] How many animals of the species were released into the preserve ?

$p(0) = \dfrac{1320}{1+10} = 120$ ANIMALS

[b] When did the population reach 792 animals ?
Find the exact answer, as well as a numerical answer rounded to 4 decimal places.

$$\frac{1320}{1+10e^{-0.9t}} = 792$$
$$1320 = 792(1+10e^{-0.9t})$$
$$\frac{5}{3} = 1 + 10e^{-0.9t}$$
$$\frac{2}{3} = 10e^{-0.9t}$$
$$\frac{1}{15} = e^{-0.9t}$$
$$\ln\frac{1}{15} = -0.9t$$
$$t = -\frac{1}{0.9}\ln\frac{1}{15}$$
$$= 3.0089$$

MONTHS LATER

The number of bacteria in a culture fits the exponential growth model. The culture originally contained 2315 bacteria, and doubles every 4 hours.

SCORE: _____ / 21 PTS

[a] Find a model for the number of bacteria after t hours.

$$p(t) = ae^{bt}$$
$$p(0) = 2315 = a \longrightarrow p(t) = 2315e^{bt}$$
$$p(4) = 2(2315) = 2315e^{4b}$$
$$2 = e^{4b}$$
$$\ln 2 = 4b$$
$$b = \tfrac{1}{4}\ln 2 = 0.1733$$

$$p(t) = 2315e^{0.1733t}$$

[b] How many bacteria will there be after 7 hours ?

$p(7) = 2315e^{0.1733(7)} \approx 7787$ BACTERIA