

Using the procedure in the Rational Function Graphing Handout posted on my website on Thu May 31, graph the following function. You must show ALL STEPS & ALL WORK.

SCORE: \_\_\_\_ / 6 PTS

Show all relevant features on your final graph (ie. your graph should not resemble a graphing calculator's graph.)

For step [6], you must calculate EXACTLY what functions  $f(x)$  resembles at each  $x$ -intercept and vertical asymptote, including finding the EXACT formulae (DO NOT use the  $k$  shorthand shown in lecture) and drawing the small pieces of graph next to those formulae. (Refer to the examples in the handout.)

$$f(x) = \frac{2x^2 + 2x}{x^3 + x^2 - 12x} = \frac{2x(x+1)}{x(x+4)(x-3)}$$

STEP [1]  $x(x+4)(x-3) \neq 0$

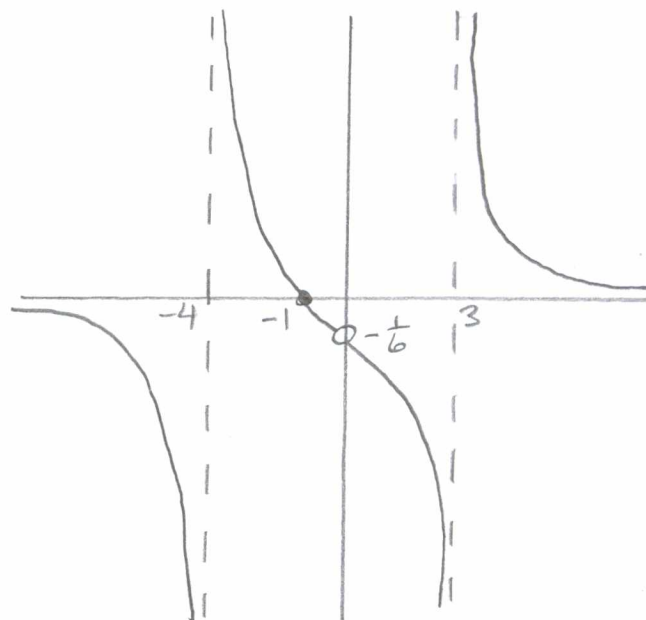
DOMAIN =  $\{x \neq 0 \text{ AND } x \neq -4 \text{ AND } x \neq 3\}$

STEP [2]  $2x(x+1) = 0$   
 $x = 0$  OR  $x = -1$        $x$ -INT =  $(-1, 0)$   
 $\uparrow$  NOT IN DOMAIN

STEP [3]  $f(0)$  DOES NOT EXIST  
 NO  $y$ -INT

STEP [4] DEGREE OF NUMERATOR  
 $<$  DEGREE OF DENOMINATOR  
 AS  $x \rightarrow \pm\infty$ ,  $y \rightarrow 0$   
 HORIZONTAL ASYMPTOTE  
 $y = 0$

GRAPH



STEP [5] SIMPLIFIED  $f(x) = \frac{2(x+1)}{(x+4)(x-3)}$        $(x+4)(x-3) = 0$   
 $x = -4$  AND  $x = 3$   
 VERTICAL ASYMPTOTES  
 HOLE AT  $x = 0$ ,  $y = \frac{2(0+1)}{(0+4)(0-3)} = -\frac{1}{6}$

STEP [6]

AT  $x = -1$ ,  $f(x) \approx \frac{2(x+1)}{(-1+4)(-1-3)} = -\frac{1}{6}(x+1)$   $\rightarrow$

AT  $x = -4$ ,  $f(x) \approx \frac{2(-4+1)}{(x+4)(-4-3)} = \frac{6}{7(x+4)}$   $\uparrow$

AT  $x = 3$ ,  $f(x) \approx \frac{2(3+1)}{(3+4)(x-3)} = \frac{8}{7(x-3)}$   $\uparrow$

Show all relevant features on your final graph (ie. your graph should not resemble a graphing calculator's graph.)

For step [6], you must calculate EXACTLY what functions  $f(x)$  resembles at each  $x$ -intercept and vertical asymptote, including finding the EXACT formulae (DO NOT use the  $k$  shorthand shown in lecture) and drawing the small pieces of graph next to those formulae. (Refer to the examples in the handout.)

$$f(x) = \frac{(x-1)(x+2)(x-3)}{(x+6)^2}$$

STEP [1]  $(x+6)^2 \neq 0$

DOMAIN =  $\{x \neq -6\}$

STEP [2]  $(x-1)(x+2)(x-3) = 0$

$x = 1$  or  $x = -2$  or  $x = 3$

$x$ -INT =  $(1, 0), (-2, 0), (3, 0)$

STEP [3]  $f(0) = \frac{(-1)(2)(-3)}{6^2} = \frac{1}{6}$

$y$ -INT =  $(0, \frac{1}{6})$

STEP [4] DEGREE OF NUMERATOR

= 1 + DEGREE OF DENOMINATOR

$(x-1)(x+2)(x-3) = x^3 - 2x^2 - 5x + 6$

$x^2 + 12x + 36 \mid x^3 - 2x^2 - 5x + 6$

$x^3 + 12x^2 + 36x$

$-14x^2 - 41x + 6$

$-14x^2 - 168x - 504$

$127x + 510$

SLANT  
ASYMPTOTE

$y = x - 14$

STEP [5]  $(x+6)^2 = 0$

$x = -6$  VERTICAL ASYMPTOTE

NO HOLES

STEP [6]

AT  $x = 1$ ,  $f(x) \approx \frac{(x-1)(1+2)(1-3)}{(1+6)^2} = -\frac{6}{49}(x-1)$   $\rightarrow$

AT  $x = -2$ ,  $f(x) \approx \frac{(-2-1)(x+2)(-2-3)}{(-2+6)^2} = \frac{15}{16}(x+2)$   $\rightarrow$

AT  $x = 3$ ,  $f(x) \approx \frac{(3-1)(3+2)(x-3)}{(3+6)^2} = \frac{10}{81}(x-3)$   $\rightarrow$

AT  $x = -6$ ,  $f(x) \approx \frac{(-6-1)(-6+2)(-6-3)}{(x+6)^2} = \frac{-252}{(x+6)^2}$   $\nearrow \searrow$

GRAPH

