Section 2.3

An argument is a sequence of statements.
The last statement is called the conclusion.
All the preceding statements are called the assumptions or hypotheses.
(The conclusion usually starts with "Therefore" or "So".)

An argument is valid if and only if ⬅ **THIS IS THE DEFINITION**
the conclusion is true in all cases where the assumptions are true.
So, an argument is invalid if and only if
there is a situation in which the assumptions are true, but the conclusion is false.

The validity of an argument is NOT simply determined by whether the conclusion is true or false by itself.

To determine if an argument is valid or invalid, ⬅ **THIS IS NOT THE DEFINITION, JUST AN**
construct a truth table with columns for each assumption and the conclusion. **ALGORITHM TO**
Highlight all rows in which all assumptions are true. (These are called the critical rows.) **CHECK IF**
If the conclusion is true in all those rows, the argument is valid. **AN ARGUMENT IS VALID**

A valid argument is a logical process by which we reach a conclusion.
The process itself assumes correct inputs (true statements),
and gives correct outputs (true conclusions).
If the process is given incorrect inputs (false statements),
there is no guarantee whether the outputs are correct or not.

A computer science parallel for the difference between
a valid argument versus a true conclusion:

Suppose a class is asked to write a program to read an arithmetic expression
of the form "a ⊗ b" (where a and b are numbers and ⊗ is either +, –, × or ÷)
from a user's input, and evaluate the expression.

John writes the program so that it simply says "1" no matter what the user inputs.
He runs a test with the inputs "6 – 5", "9 ÷ 9", "2 × 0.5" and "–7 + 8".
The answers are all correct.
This is parallel to an invalid argument (the program)
with a true conclusion (the answers).

Jane writes the program to work exactly as required if the user inputs a correct
arithmetic expression.
She runs the test with the input "Dog ^ Cat".
Her program crashes.
This is parallel to a valid argument (the program)
with a false conclusion (the crash) due to a false assumption (the input).

USE THE TRUTH TABLE ALGORITHM ABOVE TO PROVE
WHETHER THE FOLLOWING COMMON ARGUMENT FORMS ARE VALID OR INVALID

ARGUMENT #0 **(example of invalid argument with true assumptions and conclusion)**

	INVALID	**CONVERSE ERROR**
If today is Wednesday, then Math 22 meets today.		$p \rightarrow q$
Math 22 meets today.		q
Therefore, today is Wednesday.		$\therefore p$

ARGUMENT #1 **(example of valid argument with false assumptions and conclusion)**

VALID **MODUS PONENS**

If today is Wednesday, then this is Math 43. $p \rightarrow q$
Today is Wednesday. p
Therefore, **this is Math 43**. $\therefore q$

ARGUMENT #2

VALID **MODUS TOLLENS**

If you study hard, then you will do well. $p \rightarrow q$
You didn't do well. $\sim q$
Therefore, **you didn't study hard**. $\therefore \sim p$

ARGUMENT #3

VALID **GENERALIZATION**

Today is Wednesday. p
Therefore, **today is Wednesday** or this is Math 43. $\therefore p \vee q$
NOTE: **Alternate/parallel version of GENERALIZATION argument form exists using assumption "q" instead of "p"**

ARGUMENT #4

VALID **SPECIFICATION**

Today is Wednesday and this is Math 22. $p \wedge q$
Therefore, **today is Wednesday**. $\therefore p$
NOTE: **Alternate/parallel version of SPECIFICATION argument form exists using conclusion "q" instead of "p"**

ARGUMENT #5

VALID **CONJUNCTION**

Today is Wednesday. p
This is Math 22. q
Therefore, **today is Wednesday and this is Math 22**. $\therefore p \wedge q$

ARGUMENT #6

VALID **ELIMINATION**

Today is Wednesday or this is Math 43. $p \vee q$
This is not Math 43. $\sim q$
Therefore, **today is Wednesday**. $\therefore p$
NOTE: **Alternate/parallel version of ELIMINATION argument form exists using assumption "~p" instead of "~q" and conclusion "q" instead of "p"**

ARGUMENT #7

VALID **TRANSITIVITY**

If today is Wednesday, then Math 22 meets today. $p \rightarrow q$
If Math 22 meets today, then I cannot sleep in until noon. $q \rightarrow r$
Therefore, **if today is Wednesday, then I cannot sleep in until noon.** $\therefore p \rightarrow r$

ARGUMENT #8

VALID **DIVISION INTO CASES**

Today is Wednesday or today is Thursday. $p \vee q$
If today is Wednesday, then Math 22 meets today. $p \rightarrow r$
If today is Thursday, then Math 22 meets today. $q \rightarrow r$
Therefore, **Math 22 meets today**. $\therefore r$

ARGUMENT #9 **(example of invalid argument with true assumptions and false conclusion)**

INVALID **INVERSE ERROR**

If today is Tuesday, then Math 22 meets today. **$p \rightarrow q$**
Today is not Tuesday. **$\sim p$**
Therefore, Math 22 does not meet today. **$\therefore \sim q$**

NOTE: false conclusion does not automatically mean argument is invalid, even in this case

"ARGUMENT" # 10 **(example of argument for which you must determine conclusion using valid argument forms – too big for truth table algorithm anyway)**
TRY THIS YOURSELF FIRST – WILL USE AS EXAMPLE IN LECTURE

Someone killed Mr Body using a weapon. Assuming all the following statements are true, can you determine who the killer was and what weapon they used ?

If Miss Plum did not use the gun, then Mrs Red did not use the knife and Ms Scarlet did not use the wrench.
Either Mr Violet did not use the hammer or Mr Yellow used the poison.
If Mr White did not use the candlestick, then Ms Scarlet used the wrench.
If Mr Violet did not use the hammer, then Miss Plum did not use the gun.
Mr Yellow did not use the poison.

NOTE:

The argument
Statement 1
Statement 2
Statement 3 …
Statement n
Conclusion
is valid if and only if the compound statement
$(\text{Statement 1} \wedge \text{Statement 2} \wedge \text{Statement 3} \wedge \ldots \wedge \text{Statement n}) \rightarrow \text{Conclusion}$
is a **tautology**.