Write the **formal definition** of a function used in discrete math. Use correct English and mathematical notation. **SCORE: ___ / 8 PTS**

A RELATION R FROM SET A TO SET B IS A FUNCTION
IF AND ONLY IF
$\forall x \in A, \exists y \in B : (x,y) \in R$
$\wedge \forall x \in A, \forall y,z \in B, [(x,y) \in R \text{ AND } (x,z) \in R] \rightarrow y = z$

The following table shows the classes taken by 3 students during 3 quarters. **SCORE: ___ / 18 PTS**

	F10	**W11**	**S11**
Ro	Chem	Psych	Math
Sue	Chem & Math		Econ
Tri	Chem	Psych & Econ	Math

Let S = set of students = $\{Ro, Sue, Tri\}$.
Let Q = set of quarters = $\{F10, W11, S11\}$.
Let C = set of classes = $\{Psych, Chem, Econ, Math\}$.

[a] Write the negation of "$\exists c \in C : \forall s \in S, s$ took c"

[i] symbolically

$\forall c \in C, \exists s \in S :$ s DID NOT TAKE c

[ii] in English **without using variables**

FOR EVERY CLASS, THERE WAS A STUDENT WHO DIDN'T TAKE IT

OR NO STUDENT TOOK EVERY CLASS

[b] Determine if the following statement is true or false. **Justify your answers with RELEVANT examples and/or counterexamples.**

$\forall s \in S, \exists q \in Q : s$ took Math during q

$s = $ RO $\exists q \in Q :$ RO TOOK MATH DURING q

(T: S11)

$s = $ SUE SUE (T: F10)

$s = $ TRI TRI (T: S11)

TRUE

Is the following statement a tautology, a contradiction or neither ? **SCORE: ___ / 18 PTS**

<u>If it is neither, determine what simplified statement it is logically equivalent to.</u>

$$(p \oplus q) \leftrightarrow (p \rightarrow \sim q)$$

p	q	$p \oplus q$	$\sim q$	$p \rightarrow \sim q$	$(p \oplus q) \leftrightarrow (p \rightarrow \sim q)$
T	T	F	F	F	T
T	F	T	T	T	T
F	T	T	F	T	T
F	F	F	T	T	F

NEITHER,

EQUIVALENT TO $p \vee q$

Consider the statement "Students less than 15 years old must have their parents' permission." **SCORE: ___ / 14 PTS**

[a] Write the statement symbolically using **<u>exactly one quantifier</u>** and **<u>one conditional</u>**. Identify your domain and predicates clearly.

D = { STUDENTS } $\qquad \forall x \in D,\ P(x) \rightarrow Q(x)$

$P(x)$ = "x IS LESS THAN 15 YEARS OLD"

$Q(x)$ = "x HAS THEIR PARENTS' PERMISSION"

[b] Write the inverse of the statement as an English sentence without using any symbols or variables.

$\forall x \in D,\ \sim P(x) \rightarrow \sim Q(x)$

STUDENTS WHO ARE AT LEAST 15 YEARS OLD DO NOT NEED THEIR PARENTS' PERMISSION

[c] Write the contrapositive of the statement as an English sentence without using any symbols or variables.

$\forall x \in D,\ \sim Q(x) \rightarrow \sim P(x)$

STUDENTS WHO DON'T HAVE THEIR PARENTS' PERMISSION MUST BE AT LEAST 15 YEARS OLD.

Use truth tables to determine if the following argument form is valid or invalid. **Show all entries on all rows. Mark the critical rows clearly, and state whether the argument is valid or invalid.** **SCORE: ___ / 20 PTS**

If Mo is a computer science major or Mo is a math major, then Math 22 is on Mo's required course list.
Math 22 is on Mo's required course list and Mo is not a math major.
Therefore, Mo is a computer science major.

$(p \vee q) \rightarrow r$

$r \wedge \sim q$

$\therefore p$

p	q	r	$p \vee q$	$(p \vee q) \rightarrow r$	$\sim q$	$r \wedge \sim q$	p
T	T	T	T	T	F	F	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	F	F	F
F	T	F	T	F	F	F	F
F	F	T	F	T	T	T	F ← INVALID
F	F	F	F	T	T	F	F

Let $P(x) =$ "x is prime". Let $Q(x) =$ "$7x \leq x^2$". **SCORE: ___ / 12 PTS**

Let $D = \{2, 4, 5, 7, 11\}$ be the domain of both predicates.

Is the statement "$Q(x) \Rightarrow P(x)$" true or false ? **Justify your answer using truth sets.**

TRUTH SET OF $P(x) = \{2, 5, 7, 11\}$

$Q(x) = \{7, 11\}$

TRUTH SET OF $Q(x) \subseteq$ TRUTH SET OF $P(x)$

SO $Q(x) \Rightarrow P(x)$

Consider the following statement. **SCORE: ___ / 10 PTS**

"A page fault occurs only if the data is not in physical memory." IF (PAGE FAULT), THEN (DATA)

(Write your final answers in complete sentences without using any symbols or variables.)

[a] Write a logically equivalent statement using "is necessary for", **without using "if"**.

DATA NOT BEING IN PHYSICAL MEMORY IS NECESSARY FOR A PAGE FAULT TO OCCUR

[b] Write the negation of the original statement.

A PAGE FAULT OCCURS AND THE DATA IS IN PHYSICAL MEMORY

Fill in the blanks. Your answers must be in English, **not symbols**, unless explicitly stated otherwise. **SCORE: ___ / 18 PTS**

[a] The argument

"If Jennifer receives a scholarship from Stanford, then she will enroll at Stanford in the fall.

Jennifer enrolled at Stanford in the fall.

Therefore, she received a scholarship from Stanford."

is an example of CONVERSE ERROR.

[b] T is a **proper** subset of S if and only if

EVERY ELEMENT OF T IS AN ELEMENT OF S AND S CONTAINS AT LEAST ONE ELEMENT NOT IN T.

[c] A conditional statement is logically equivalent to its CONTRAPOSITIVE.

[d] $A \times B$, which is read as "A CROSS B", is called the CARTESIAN PRODUCT OF A AND B

In **set builder** notation (ie. symbolically), $A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$.

[e] In the conditional "$g \rightarrow h$",

h is called the CONCLUSION and g is called the HYPOTHESIS.

[f] The CONJUNCTION OF V AND W is denoted by $v \wedge w$,

which is read as "v and w".

[g] An argument (or argument form) is valid if and only if

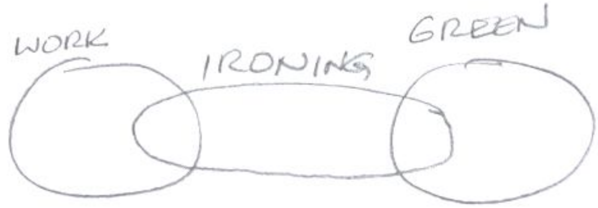
ITS CONCLUSION IS TRUE IN ALL SITUATIONS WHERE ITS PREMISES ARE TRUE.

⬆ Your answer for [g] must NOT use the terms "truth table" or "critical row".

Indicate whether the following argument is valid or invalid. **Support your answer using a diagram.** **SCORE: ___ / 12 PTS**

None of my work shirts are green.
Some of my green shirts need ironing.
Therefore, none of my work shirts need ironing.

INVALID



RULES OF INFERENCE			**Contradiction (CONT)**	$\sim p \to \mathbf{c}$ $\therefore p$	
Modus Ponens (MP)	$p \to q$ p $\therefore q$		**Modus Tollens (MT)**	$p \to q$ $\sim q$ $\therefore \sim p$	
Generalization (GEN)	p $\therefore p \vee q$	q $\therefore p \vee q$	**Specialization (SPEC)**	$p \wedge q$ $\therefore p$	$p \wedge q$ $\therefore q$
Elimination (ELIM)	$p \vee q$ $\sim p$ $\therefore q$	$p \vee q$ $\sim q$ $\therefore p$	**Conjunction (CONJ)**	p q $\therefore p \wedge q$	
Transitivity (TRAN)	$p \to q$ $q \to r$ $\therefore p \to r$		**Division into Cases (CASE)**	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$	

Use the Rules of Inference to deduce the conclusion from the hypotheses. **SCORE: ___ / 20 PTS**
State the rules used (or "GIVEN") for each step. You may use the abbreviations in the table above.
Do NOT rewrite any of the hypotheses using logical equivalences.

$\sim w \to k$
$\sim m \vee h$
$b \wedge w \to m$
$\sim k \wedge b$
$\therefore h$

$\sim k \wedge b$ GIVEN
$\therefore \sim k$ SPEC
$\therefore b$ SPEC
$\sim w \to k$ GIVEN
$\therefore w$ MT
$\therefore b \wedge w$ CONJ
$b \wedge w \to m$ GIVEN
$\therefore m$ MP
$\sim m \vee h$ GIVEN
$\therefore h$ ELIM