

Use truth tables to determine if the following argument form is valid or invalid. Show all entries on all rows.  
Mark the critical rows clearly, and state whether the argument is valid or invalid.

SCORE: \_\_\_ / 20 PTS

If Mo is a computer science major or Mo is a math major, then Math 22 is on Mo's required course list.  
 Math 22 is on Mo's required course list and Mo is not a math major.  
 Therefore, Mo is a computer science major.

$$(p \vee q) \rightarrow r$$

$$r \wedge \sim q$$

$$\therefore p$$

P	q	r	$p \vee q$	$(p \vee q) \rightarrow r$	$\sim q$	$r \wedge \sim q$	P
T	T	T	T	T	F	F	T
T	T	F	T	F	F	F	T
T	F	T	T	<u>T</u>	T	<u>T</u>	<u>T</u>
T	F	F	T	F	T	F	T
F	T	T	T	T	F	F	F
F	T	F	T	F	F	F	F
F	F	T	F	<u>F</u>	T	<u>T</u>	<u>F</u>
F	F	F	F	T	T	F	F

INVALID

Let  $P(x) = "x \text{ is prime}"$ . Let  $Q(x) = "7x \leq x^2"$ .

SCORE: \_\_\_ / 12 PTS

Let  $D = \{2, 4, 5, 7, 11\}$  be the domain of both predicates.

Is the statement " $Q(x) \Rightarrow P(x)$ " true or false? Justify your answer using truth sets.

$$\text{TRUTH SET OF } P(x) = \{2, 5, 7, 11\}$$

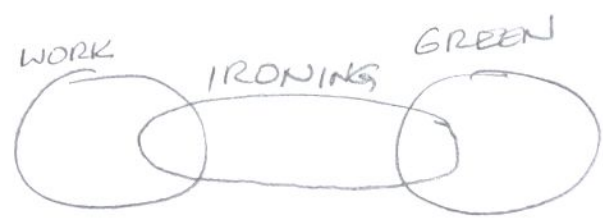
$$Q(x) = \{7, 11\}$$

$$\text{TRUTH SET OF } Q(x) \subseteq \text{TRUTH SET OF } P(x)$$

$$\text{SO } Q(x) \Rightarrow P(x)$$

None of my work shirts are green.  
Some of my green shirts need ironing.  
Therefore, none of my work shirts need ironing.

INVALID



RULES OF INFERENCE		Contradiction (CONT)	$\sim p \rightarrow c$ $\therefore p$
Modus Ponens (MP)	$p \rightarrow q$ $p$ $\therefore q$	Modus Tollens (MT)	$p \rightarrow q$ $\sim q$ $\therefore \sim p$
Generalization (GEN)	$p$ $\therefore p \vee q$	Specialization (SPEC)	$p \wedge q$ $\therefore p$
Elimination (ELIM)	$p \vee q$ $\sim p$ $\therefore q$	Conjunction (CONJ)	$p$ $q$ $\therefore p \wedge q$
Transitivity (TRAN)	$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	Division into Cases (CASE)	$p \vee q$ $p \rightarrow r$ $q \rightarrow r$ $\therefore r$

Use the Rules of Inference to deduce the conclusion from the hypotheses.  
State the rules used (or "GIVEN") for each step. You may use the abbreviations in the table above.  
Do NOT rewrite any of the hypotheses using logical equivalences.

SCORE: \_\_\_\_ / 20 PTS

$\sim w \rightarrow k$   
 $\sim m \vee h$   
 $b \wedge w \rightarrow m$   
 $\sim k \wedge b$   
 $\therefore h$

$\sim k \wedge b$  GIVEN  
 $\therefore \sim k$  SPEC  
 $\therefore b$  SPEC  
 $\sim w \rightarrow k$  GIVEN  
 $\therefore w$  MT  
 $\therefore b \wedge w$  CONJ  
 $b \wedge w \rightarrow m$  GIVEN  
 $\therefore m$  MP  
 $\sim m \vee h$  GIVEN  
 $\therefore h$  ELIM

Consider the following statement.

SCORE: \_\_\_ / 10 PTS

"A page fault occurs only if the data is not in physical memory."

IF (PAGE FAULT), THEN (DATA)

(Write your final answers in complete sentences without using any symbols or variables.)

- [a] Write a logically equivalent statement using "is necessary for", without using "if".

DATA NOT BEING IN PHYSICAL MEMORY  
IS NECESSARY FOR A PAGE FAULT TO OCCUR

- [b] Write the negation of the original statement.

A PAGE FAULT OCCURS AND  
THE DATA IS IN PHYSICAL MEMORY

Fill in the blanks. Your answers must be in English, not symbols, unless explicitly stated otherwise.

SCORE: \_\_\_ / 18 PTS

- [a] The argument

"If Jennifer receives a scholarship from Stanford, then she will enroll at Stanford in the fall.

Jennifer enrolled at Stanford in the fall.

Therefore, she received a scholarship from Stanford."

is an example of CONVERSE ERROR.

- [b]  $T$  is a proper subset of  $S$  if and only if

EVERY ELEMENT OF  $T$  IS AN ELEMENT OF  $S$  AND  
 $S$  CONTAINS AT LEAST ONE ELEMENT NOT IN  $T$ .

- [c] A conditional statement is logically equivalent to its CONTRAPOSITIVE.

- [d]  $A \times B$ , which is read as " $A$  CROSS  $B$ ", is called the CARTESIAN PRODUCT OF  $A$  AND  $B$

In set builder notation (ie. symbolically),  $A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$ .

- [e] In the conditional " $g \rightarrow h$ ",

$h$  is called the CONCLUSION and  $g$  is called the HYPOTHESIS.

- [f] The CONJUNCTION OF  $V$  AND  $W$  is denoted by  $v \wedge w$ ,  
which is read as " $v$  and  $w$ ".

- [g] An argument (or argument form) is valid if and only if

ITS CONCLUSION IS TRUE IN ALL SITUATIONS  
WHERE ITS PREMISES ARE TRUE.

**↑ Your answer for [g] must NOT use the terms "truth table" or "critical row".**

Is the following statement a tautology, a contradiction or neither?

SCORE: \_\_\_ / 18 PTS

If it is neither, determine what simplified statement it is logically equivalent to.

$$(p \oplus q) \leftrightarrow (p \rightarrow \sim q)$$

$p$	$q$	$p \oplus q$	$\sim q$	$p \rightarrow \sim q$	$(p \oplus q) \leftrightarrow (p \rightarrow \sim q)$
T	T	F	F	F	T
T	F	T	T	T	T
F	T	T	F	T	T
F	F	F	T	T	F

NEITHER,

EQUIVALENT TO  $p \vee q$

Consider the statement "Students less than 15 years old must have their parents' permission."

SCORE: \_\_\_ / 14 PTS

- [a] Write the statement symbolically using exactly one quantifier and one conditional.  
Identify your domain and predicates clearly.

$$D = \{\text{STUDENTS}\}$$

$$\forall x \in D, P(x) \rightarrow Q(x)$$

$$P(x) = "x \text{ IS LESS THAN 15 YEARS OLD}"$$

$$Q(x) = "x \text{ HAS THEIR PARENTS' PERMISSION}"$$

- [b] Write the inverse of the statement as an English sentence without using any symbols or variables.

$$\forall x \in D, \sim P(x) \rightarrow \sim Q(x)$$

STUDENTS WHO ARE AT LEAST 15 YEARS  
OLD DO NOT NEED THEIR PARENTS' PERMISSION

- [c] Write the contrapositive of the statement as an English sentence without using any symbols or variables.

$$\forall x \in D, \sim Q(x) \rightarrow \sim P(x)$$

STUDENTS WHO DON'T HAVE THEIR PARENTS'  
PERMISSION MUST BE AT LEAST 15 YEARS OLD.



Write the **formal definition** of a function used in discrete math. Use correct English and mathematical notation. SCORE: \_\_\_\_ / 8 PTS

A RELATION  $R$  FROM SET  $A$  TO SET  $B$  IS A FUNCTION  
IF AND ONLY IF  
 $\forall x \in A, \exists y \in B: (x, y) \in R$   
 $\wedge \forall x \in A, \forall y, z \in B, [(x, y) \in R \text{ AND } (x, z) \in R] \rightarrow y = z$

The following table shows the classes taken by 3 students during 3 quarters. SCORE: \_\_\_\_ / 18 PTS

	F10	W11	S11
Ro	Chem	Psych	Math
Sue	Chem & Math		Econ
Tri	Chem	Psych & Econ	Math

Let  $S$  = set of students =  $\{Ro, Sue, Tri\}$ .  
Let  $Q$  = set of quarters =  $\{F10, W11, S11\}$ .  
Let  $C$  = set of classes =  $\{Psych, Chem, Econ, Math\}$ .

[a] Write the negation of " $\exists c \in C: \forall s \in S, s$  took  $c$ "

[i] symbolically

$\forall c \in C, \exists s \in S: s \text{ DID NOT TAKE } C$

[ii] in English **without using variables**

FOR EVERY CLASS, THERE WAS A STUDENT WHO DIDN'T  
TAKE IT

OR NO STUDENT TOOK EVERY CLASS

[b] Determine if the following statement is true or false. **Justify your answers with RELEVANT examples and/or counterexamples.**

$\forall s \in S, \exists q \in Q: s$  took Math during  $q$

$S = RO \quad \exists q \in Q: RO$  TOOK MATH DURING  $q$

$S = SUE$

SUE

(T: S11)

$S = TRI$

TRI

(T: F10)

(T: S11)

TRUE