Show that the function $f : Z^+ \times Z^+ \to Z^+$ defined by $f(x, y) = x + y$ is **NOT** onto. **SCORE: ___ / 8 PTS**

$1 \in \mathbb{Z}^+$

BUT SINCE $x, y \in \mathbb{Z}^+$, $x \geq 1$ AND $y \geq 1$

SO $f(x,y) = x + y \geq 2$

SO $f(x,y) \neq 1$ FOR ANY $(x,y) \in \mathbb{Z}^+ \times \mathbb{Z}^+$

Find the error in the following proof. **SCORE: ___ / 7 PTS**

"Theorem": If R is a binary relation on set $A \subseteq U$, and R is symmetric and transitive, then R is reflexive.

"Proof": Let $A \subseteq U$ and let R be a binary relation on set A such that R is symmetric and transitive.
By symmetry, xRy implies yRx.
Since xRy and yRx, by transitivity, xRx.
Therefore, by definition of reflexive, R is reflexive.

NOTE: The "theorem" is a false statement. Do NOT attempt to find a counterexample to the "theorem".

IF THERE IS AN ELEMENT $x \in A$
WHICH IS NOT RELATED TO ANY ELEMENT,
THEN THE SUPPOSITION xRy IS FALSE

SEE E-MAIL FROM MAR 11

Let $A_5 = \{1, 2, 3, 4, 5\}$, $A_7 = \{1, 2, 3, 4, 5, 6, 7\}$ and $A_9 = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. **SCORE: ___ / 15 PTS**

[a] How many one-to-one functions are there with domain A_9 and co-domain A_7 ?

0

[b] How many one-to-one functions are there with domain A_5 and co-domain A_7 ?

SEE 9.2 #22

1 SELECT A VALUE FOR $f(1)$ # WAYS

2 $f(2)$ - NOT 1 $= 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$

3 $f(3)$ - NOT 1, 2 OR

4 $f(4)$ - NOT 1, 2, 3 $P(7,5)$

5 $f(5)$ - NOT 1, 2, 3, 4

WRITE A FORMAL PROOF that the function $f : R^+ \to R$ defined by $f(x) = x^2 + x$ is one-to-one. **SCORE: ___ / 20 PTS**

NOTE: Do NOT use calculus, graphs or the horizontal line test.

LET $x, y \in \mathbb{R}^+$ SUCH THAT $f(x) = f(y)$

SO $x^2 + x = y^2 + y$

SO $x^2 - y^2 + x - y = 0$

SO $(x+y)(x-y) + (x-y) = 0$

SO $(x+y+1)(x-y) = 0$

SINCE $x \geq 0$ AND $y \geq 0$

THEREFORE $x+y+1 \geq 1$ IE. $x+y+1 \neq 0$

SO $x - y = 0$ BY ZERO PRODUCT PROPERTY

SO $x = y$

SO f IS 1-1 BY DEF'N OF 1-1

SEE 7.2 LECTURE NOTES

Let R be an equivalence relation on set A. **WRITE A FORMAL PROOF** for the following statement. **SCORE: ___ / 30 PTS**

Use the definitions in sections 8.2 and 8.3 but do NOT use any of the lemmas, theorems or homework exercises as justification.

For all $a, b, c \in A$, if $a \in [c]$ and $b \in [c]$, then $[a] = [b]$.

LET $a, b, c \in A$ SUCH THAT $a \in [c]$ AND $b \in [c]$

LET $x \in [a]$

SO xRa BY DEF'N OF [ ]

AND aRc AND bRc BY DEF'N OF [ ]

SO cRb BY SYMMETRY

SINCE xRa, aRc AND cRb,

BY TRANSITIVITY, xRb

SO $x \in [b]$ BY DEF'N OF [ ]

SO $[a] \subseteq [b]$ BY DEF'N OF $\subseteq$

LET $x \in [b]$

SO xRb

AND aRc AND bRc BY DEF'N OF [ ]

SO cRa BY SYMMETRY

SINCE xRb, bRc AND cRa,

BY TRANSITIVITY, xRa

SO $x \in [a]$ BY DEF'N OF [ ]

SO $[b] \subseteq [a]$ BY DEF'N OF $\subseteq$

SO $[a] = [b]$ BY DEF'N OF =

Let M be the binary relation on Z^+ defined by **SCORE: ___ / 25 PTS**

xMy if and only if there exists a prime number p such that $p \mid x$ and $p \mid y$

SEE 8.2 #17

Determine if M is an equivalence relation.
Give a brief justification (**<u>NOT</u>** a formal proof) for each property in the definition of an equivalence relation which M satisfies.
Give a counterexample for each property in the definition of an equivalence relation which M does not satisfy.
<u>You may use theorems from previous chapters as justification. If you don't know the name of a theorem, summarize what it says.</u>

$1 \not M 1$ SINCE THERE IS NO PRIME p SUCH THAT $p \mid 1$ AND $p \mid 1$
SO M IS NOT REFLEXIVE

IF xMy, THERE IS A PRIME p SUCH THAT $p \mid x$ AND $p \mid y$
SO $p \mid y$ AND $p \mid x$
SO yMx

SO M IS SYMMETRIC

$2M6$ SINCE 2 IS A PRIME AND $2 \mid 2$ AND $2 \mid 6$
$6M9$ 3 $3 \mid 6$ $3 \mid 9$
$2 \not M 9$ SINCE THERE IS NO PRIME p SUCH THAT $p \mid 2$ AND $p \mid 9$
SO M IS NOT TRANSITIVE M IS NOT AN EQ. REL'N

Eight friends (Chris, Terry, Dana, Pat, Jess, Bailey, Taylor & Reese) are at Great America. **SCORE: ___ / 18 PTS**
There are 5 adjacent empty seats on the FireFall thrill ride. Chris will only get on the ride if seated directly next to Terry.
How many ways can 5 of the friends fill those seats ?

IF CHRIS DOES NOT RIDE, # WAYS = $P(7,5)$

IF CHRIS RIDES,
1. CHOOSE 3 MORE FRIENDS (+ TERRY)
2. PERMUTE EVERYONE BUT CHRIS
3. PUT CHRIS EITHER ON TERRY'S LEFT OR RIGHT

WAYS $= C(6,3) \cdot P(4,4) \cdot 2$

$P(7,5) + 2C(6,3)P(4,4)$

A group of 12 people decide to play volleyball, in 2 teams of 6 players each. Ty and Rex decide to bet on who will win, so they must not be on the same team. How many ways are there to split the 12 people into 2 teams ? **SCORE: ___ / 12 PTS**

SELECT 5 PEOPLE FOR TY'S TEAM
(ALL OTHERS GO ON REX'S TEAM)

$C(10,5)$

How many 5 card poker hands have exactly one pair – that is, two cards of one value, and three other cards all of different values (from each other as well as from the first two cards) ? 9♣ 9♦ 2♦ 7♥ K♠ would be an example. **SCORE: ___ / 15 PTS**

1. SELECT A VALUE FOR PAIR
2. 2 SUITS ″
3. 3 VALUES (NOT 1)
4. SUIT FOR LOWEST VALUE IN 3
5. 2ND LOWEST
6. HIGHEST

SEE 9.5 #11

\# WAYS

$= 13 \cdot C(4,2) \cdot C(12,3) \cdot 4 \cdot 4 \cdot 4$