Fri Jan 13, 2012

SCORE: ___ / 20 POINTS

Let $F = \{-1, 0, 1\}$.
Let $G = \{0, 1, 2\}$.

$F \times G = \{ (-1,0), (-1,1), (-1,2), (0,0), (0,1), (0,2), (1,0), (1,1), (1,2) \}$

SCORE: 5 / 5 POINTS

Let K be the relation from F to G defined by xKy if and only if $x^2 - y^2$ is a multiple of 3.

-3
0
3
6
9

[a] Write K in set roster notation.

$K = \{ \underbrace{(-1,1)}_{\frac{1}{2}}, \underbrace{(-1,2)}_{\frac{1}{2}}, \underbrace{(0,0)}_{\frac{1}{2}}, \underbrace{(1,1)}_{\frac{1}{2}}, \underbrace{(1,2)}_{\frac{1}{2}} \}$

[b] Is K a function ? Why or why not ?

it is not. -1 has two
$(-1,1)$ and $(-1,2) \in K$ where $1 \neq 2$

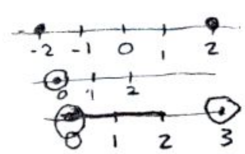
1

[c] If $H = \{3, 4\}$, write $H \times G$ in set roster notation.

$G = \{0,1,2\}$

$H \times G = \{ \underbrace{(3,0)}_{\frac{1}{4}}, \underbrace{(3,1)}_{\frac{1}{4}}, \underbrace{(3,2)}_{\frac{1}{4}}, \underbrace{(4,0)}_{\frac{1}{4}}, \underbrace{(4,1)}_{\frac{1}{4}}, \underbrace{(4,2)}_{\frac{1}{4}} \}$

Let $A = \{x \in Z \mid x^2 < 5\}$.
Let $B = \{x \in Z^{nonneg} \mid x^3 < 9\}$.
Let $C = \{x \in Z \mid 0 \le x < 3\}$.



SCORE: 2 / 4 POINTS

Are the following statements true or false ? Explain **very briefly** your answers. (No points if no explanation given.)

[a] $A = C$

False

$A = \{-2, -1, 0, 1, 2\}$

$C = \{0, 1, 2\}$

$-1 \in A$, $-1 \notin C$
$-2 \in A$, $-2 \notin C$

2

[b] B is a **proper** subset of C

$B = \{1, 2\}$

yes. $B \subseteq C$ howere there is $0 \in C$ but $0 \notin B$

CONTINUED ➡

MULTIPLE CHOICE: Which of the following statements are true ? **SCORE: 2 / 2 POINTS**

[1] $x \in \{\{x\}, y, z\}$ False
[2] $\{x\} \subseteq \{\{x\}, y, z\}$ False
[3] $\{z\} \subseteq \{\{x\}, y, z\}$ True

2

(a)	none of the above are true	(b)	all of the above are true	(c)	only [3] is true
(d)	only [1] and [2] are true	(e)	only [1] and [3] are true	(f)	only [2] and [3] are true

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Classify each statement as Universal Conditional (**UC**), Universal Existential (**UE**) or Existential Universal (**EU**). **SCORE: 1 / 2 POINTS**

[a] Some positive integer is less or equal to every positive integer. EU 1

[b] Everyone who rides the roller coaster must be at least 54 inches tall. UE

Rewrite the following statement using the formal universal existential structure mentioned in lecture. **SCORE: ___ / 3 POINTS**

NOTE: The answer requires 2 variables.
You may use algebra and/or symbolic set notation where appropriate.

"Every positive integer has a reciprocal."

For every integer x such that it is positive there is r, where $r = \frac{1}{x}$.

$\frac{1}{2}$ * $\frac{1}{2}$ * $\frac{1}{2}$

If $W = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $Y = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$, **SCORE: 1 / 1 POINTS**
how many elements are in $Y \times W$?

11×13
143 elements
1

A RELATION

Write the **formal definition** of a function used in discrete math. Use correct English and mathematical notation. **SCORE: 2 / 3 POINTS**

given two sets A and B, we say R is a function from A to B only and only if two conditions are satisfied:-

① for every x element in A ($x \in A$) there is some ~~($y \in B$)~~ such that $(x,y) \in R$
existance condition.

② for every element x in A, $x \in A$, and y in B, $y \in B$, and z in B, $z \in B$, such that $(x,y) \in R$ and $(x,z) \in R$ only and only if $z = y$ "uniqueness condition".

A is the domain
B is the codomain

[1 BONUS POINT] If the number of elements in $A \times B$ is prime, what conclusions can you draw about A and B ? 1 POINT

{( )( )} {1}

then one of them has prime number of element
and the other one has only one element.