Quiz 1 Version 1
Fri Jan 13, 2012

**SCORE:** 15 / 20 POINTS

Write the **formal definition** of a function used in discrete math. Use correct English and mathematical notation. **SCORE:** 3 / 3 POINTS

A relation R with domain A and co-domain B is a function if and only if

1) for all $x \in A$, there is a $y \in B$ such that $(x, y) \in R$

2) for all $x \in A$ and for all $y, z \in B$ if $(x, y) \in R$ and $(x, z) \in R$ then $y = z$

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If $W = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $Y = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$, how many elements are in $Y \times W$ ? **SCORE:** 1 / 1 POINTS

$11 \times 13$

$$\begin{array}{r} 11 \\ \times\ 13 \\ \hline 33 \\ 11\ \ 0 \\ \hline 143 \end{array}$$

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Classify each statement as Universal Conditional (**UC**), Universal Existential (**UE**) or Existential Universal (**EU**). **SCORE:** 1 / 2 POINTS

[a] Some positive integer is less or equal to every positive integer. UE

[b] Everyone who rides the roller coaster must be at least 54 inches tall. UC

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MULTIPLE CHOICE: Which of the following statements are true ? **SCORE:** 2 / 2 POINTS

[1] $x \in \{\{x\}, y, z\}$

[2] $\{x\} \subseteq \{\{x\}, y, z\}$

[3] $\{z\} \subseteq \{\{x\}, y, z\}$

| | | | | | |
|---|---|---|---|---|---|
| (a) | none of the above are true | (b) | all of the above are true | (c) | only [3] is true |
| (d) | only [1] and [2] are true | (e) | only [1] and [3] are true | (f) | only [2] and [3] are true |

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Rewrite the following statement using the formal universal existential structure mentioned in lecture. **SCORE:** 2 / 3 POINTS

**NOTE: The answer requires 2 variables.**
**You may use algebra and/or symbolic set notation where appropriate.**

"Every positive integer has a reciprocal."

Every positive integer $x$, has a positive integer $y$, such that $\frac{1}{x} = y$.

**CONTINUED ➡**

Let $B = \{x \in Z^{nonneg} \mid x^3 < 9\}$. {0, 1, 2}

Let $C = \{x \in Z \mid 0 \le x < 3\}$. {0, 1, 2}

Are the following statements true or false ? Explain **very briefly** your answers. (No points if no explanation given.)

[a] $A = C$ False

A ≠ C because A is a set of all integers who's square is less than 5 which would include -2, -1 where as C only contains elements which are greater or equal to 0, hence A ≠ C.

2

[b] $B$ is a **proper** subset of $C$

yes, because they both contain only 3 numbers that are present in both sets.

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Let $F = \{-1, 0, 1\}$.

Let $G = \{0, 1, 2\}$.

**SCORE: 3 / 5 POINTS**

Let $K$ be the relation from $F$ to $G$ defined by $xKy$ if and only if $x^2 - y^2$ is a multiple of 3.

[a] Write $K$ in set roster notation.

{(-1, 1), (0, 0) (1, 1)}

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

[b] Is $K$ a function ? Why or why not ?

yes, because it satisfys both condition of the function

1) Existance → there is a $y \in G$ of $x \in f$, such that $(x, y) \in k$

2) Uniqueness → $(x, y)$ & $(x, z)$ $y = z$

[c] If $H = \{3, 4\}$, write $H \times G$ in set roster notation.

{(3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2)}

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

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**[1 BONUS POINT]** If the number of elements in $A \times B$ is prime, what conclusions can you draw about $A$ and $B$ ? 1 POINT

That the no. of element. in A an B have to 1 of prime