Math 22
Quiz 1 Version A
Fri Jan 13, 2012

NAME YOU ASKED TO BE CALLED IN CLASS: _____

SCORE: ___ / 20 POINTS

Write the **formal definition** of a function used in discrete math. Use correct English and mathematical notation. **SCORE: ___ / 3 POINTS**

A relation R from set A to set B is a function if and only if

for all $x \in A$, there exists $y \in B$ such that $(x, y) \in R$

and for all $x \in A$, for all $y, z \in B$, if $(x, y) \in R$ and $(x, z) \in R$, then $y = z$

If $W = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $Y = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$, how many elements are in $Y \times W$? **SCORE: ___ / 1 POINTS**

$11 \times 13 =$ 143 **1 point**

Classify each statement as Universal Conditional (**UC**), Universal Existential (**UE**) or Existential Universal (**EU**). **SCORE: ___ / 2 POINTS**

[a] Some positive integer is less or equal to every positive integer. EU **1 point**

[b] Everyone who rides the roller coaster must be at least 54 inches tall. UC **1 point**

MULTIPLE CHOICE: Which of the following statements are true ? **SCORE: ___ / 2 POINTS**

[1] $x \in \{\{x\}, y, z\}$
[2] $\{x\} \subseteq \{\{x\}, y, z\}$
[3] $\{z\} \subseteq \{\{x\}, y, z\}$

2 points

(a) none of the above are true (b) all of the above are true (c) only [3] is true

(d) only [1] and [2] are true (e) only [1] and [3] are true (f) only [2] and [3] are true

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Rewrite the following statement using the formal universal existential structure mentioned in lecture. **SCORE: ___ / 3 POINTS**
NOTE: The answer requires 2 variables.
You may use algebra and/or symbolic set notation where appropriate.

"Every positive integer has a reciprocal."

For every | positive integer x | there is a | real number y | such that | $y = \frac{1}{x}$.

½ point ½ point ½ point ½ point ½ point ½ point

➡SUBTRACT 1 point if you wrote "there is a real number y" before "for every positive integer x"

OR

For all | $x \in Z^+$ | there exists | $y \in R$ | such that | $y = \frac{1}{x}$.

½ point ½ point ½ point ½ point ½ point ½ point

➡SUBTRACT 1 point if you wrote "there is a real number y" before "for every positive integer x"

CONTINUED ➡

Let $A = \{x \in Z \mid x^2 < 5\}$. **SCORE: ___ / 4 POINTS**

Let $B = \{x \in Z^{nonneg} \mid x^3 < 9\}$.

Let $C = \{x \in Z \mid 0 \le x < 3\}$.

Are the following statements true or false ? Explain **very briefly** your answers. (No points if no explanation given.)

[a] $A = C$

False. $-2 \in A$ but $-2 \notin C$. **2 points (no points for "FALSE" if incorrect reason given)**

OR

False. $-1 \in A$ but $-1 \notin C$. **2 points (no points for "FALSE" if incorrect reason given)**

[b] B is a **proper** subset of C

False. C does not contain any element that is not in B since $B = C = \{0, 1, 2\}$

2 points (no points for "FALSE" if incorrect reason given)

(even though every element of B is also in C).

Let $F = \{-1, 0, 1\}$. **SCORE: ___ / 5 POINTS**

Let $G = \{0, 1, 2\}$.

Let K be the relation from F to G defined by xKy if and only if $x^2 - y^2$ is a multiple of 3.

[a] Write K in set roster notation.

$\{(-1, 1), \quad (-1, 2), \quad (0, 0), \quad (1, 1), \quad (1, 2)\}$ **½ point for each ordered pair = 2½ points total**

➡SUBTRACT ½ point if not written in proper set notation

[b] Is K a function ? Why or why not ?

No. $(-1, 1) \in K$ and $(-1, 2) \in K$, but $1 \neq 2$ **1 point (no points for "NO" if incorrect reason given)**

OR

No. $(1, 1) \in K$ and $(1, 2) \in K$, but $1 \neq 2$ **1 point (no points for "NO" if incorrect reason given)**

[c] If $H = \{3, 4\}$, write $H \times G$ in set roster notation.

$\{(3, 0), \quad (3, 1), \quad (3, 2), \quad (4, 0), \quad (4, 1), \quad (4, 2)\}$ **¼ point for each ordered pair = 1½ points total**

➡SUBTRACT ½ point if not written in proper set notation