Fri Feb 3, 2012

SCORE: \_\_\_ / 30 POINTS

Suppose you wanted to prove the following statement:

SCORE: \_\_\_ / 3 POINTS

If two even integers are amicable, then their sum is perfect.

Write only the first sentence of a formal proof.

NOTE: "Amicable" and "perfect" are actual properties of numbers. You do not need to know what they mean to answer this question.

Let x and y be two particular but arbitrarily chosen even integers such that x and y are amicable.







Write a formal proof of the following statement.

SCORE: /8 POINTS

Do NOT use any properties regarding the sum, difference or product of even/odd integers mentioned in section 4.2 of your textbook.

For all even integers m, and all odd integers n,  $3m^2 - n^2$  is odd.



PROOF: Let m be a particular but arbitrarily chosen even integer, and let n be a particular but arbitrarily chosen odd integer.

By the definition of even, m = 2a for some integer  $a \cdot n$ 

By the definition of odd, n = 2b + 1 for some integer b

$$3m^2 - n^2 = 12a^2 - 4b^2 - 4b - 1 = 2(6a^2 - 2b^2 - 2b - 1) + 1$$

and  $6a^2 - 2b^2 - 2b - 1$  is an integer since Z is closed under  $\times$  and -.

By the definition of odd,  $3m^2 - n^2$  is odd.





SCORE: \_\_\_ / 2 POINTS

 $\forall x \in \mathbb{Z}, x \mid 2x$ 

The statement is false.

COUNTEREXAMPLE: x = 0, since the definition of | does not allow for n = 0 when saying  $n \mid d$ .

Is the following statement true or false? Explain your answer briefly. (You do not need to write a formal proof.)

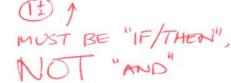




An integer n is prime if and only if n > 1 and for all positive integers r and s, if n = rs, then r = 1 or s = 1







Prove that the following statement is **false**.

For all integers m, if m is prime, then  $m^2 - 2$  is prime.

COUNTEREXAMPLE: 
$$\underline{m=11} \rightarrow \underline{m^2-2=119} = \underline{7 \times 17}$$

## [MULTIPLE CHOICE] Which of the following statements are true?

SCORE: /3 POINTS

- The set of prime numbers is closed under addition. (1)
- (2)The set of composite numbers is closed under addition.
- The set of composite numbers is closed under multiplication. (3)
- a none of the above
- [b] (2) & (3) only

[c] (1) & (3) only

[d] (1) only [e] (2) only [f](3) only

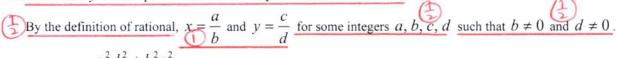
Write a formal proof of the following statement.

SCORE: \_\_\_ / 8 POINTS

Do NOT use any properties regarding the closure of the set of rational numbers.

The sum of the squares of two rational numbers is rational.

PROOF: Let x and y be two particular but arbitrarily chosen rational numbers.



$$x^{2} + y^{2} = \frac{a^{2}d^{2} + b^{2}c^{2}}{b^{2}d^{2}}, \text{ and } \underline{a^{2}d^{2} + b^{2}c^{2}}, b^{2}d^{2} \text{ are integers since } Z \text{ is closed under } \times \text{ and } +,$$
and  $b^{2}d^{2} \neq 0$  by the Zero Product Property.

and  $b^2 d^2 \neq 0$  by the Zero Product Property.

So, by the definition of rational, 
$$x^2 + y^2$$
 is rational.

