Math 22
Quiz 3 Version X
Fri Feb 3, 2012

NAMES YOU ASKED TO BE CALLED IN CLASS: ____________________

____

SCORE: ___ / 30 POINTS

Suppose you wanted to prove the following statement: **SCORE: ___ / 3 POINTS**

If the sum of two even integers is perfect, then the integers are amicable.

Write **only the first sentence** of a formal proof.
NOTE: "Amicable" and "perfect" are actual properties of numbers. You do not need to know what they mean to answer this question.

Let x and y be two particular but arbitrarily chosen even integers such that $x + y$ is perfect.

1 1 1

Write a formal proof of the following statement. **SCORE: ___ / 8 POINTS**
Do NOT use any properties regarding the closure of the set of rational numbers.

The difference of the squares of two rational numbers is rational.

PROOF: Let x and y be two particular but arbitrarily chosen rational numbers. 1

$\frac{1}{2}$ By the definition of rational, $x = \dfrac{a}{b}$ and $y = \dfrac{c}{d}$ 1 for some integers a, b, c, d $\frac{1}{2}$ such that $b \neq 0$ and $d \neq 0$. $\frac{1}{2}$

1 $x^2 - y^2 = \dfrac{a^2d^2 - b^2c^2}{b^2d^2}$, and $a^2d^2 - b^2c^2, b^2d^2$ are integers since Z is closed under $\times$ and $-$, $\frac{1}{2}$ $\frac{1}{2}$

1 and $b^2d^2 \neq 0$ by the Zero Product Property. $\frac{1}{2}$

So, by the definition of rational, $x^2 - y^2$ is rational.

$\frac{1}{2}$ $\frac{1}{2}$

Prove that the following statement is **false**. **SCORE: ___ / 3 POINTS**

For all integers m, if m is prime, then $m^2 - 2$ is prime.

COUNTEREXAMPLE: $m = 11 \rightarrow m^2 - 2 = 119 = 7 \times 17$

2 $\frac{1}{2}$ $\frac{1}{2}$

Write the formal definition of "prime". **SCORE: ___ / 3 POINTS**

An integer n is prime if and only if $n > 1$ and for all positive integers r and s, if $n = rs$, then $r = 1$ or $s = 1$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$ MUST BE "IF/THEN" NOT "AND"

Is the following statement true or false ? Explain your answer briefly. (You do not need to write a formal proof.) **SCORE: ___ / 2 POINTS**

$\forall x \in Z, \quad x \mid 2x$

The statement is false. $\frac{1}{2}$

COUNTEREXAMPLE: $x = 0$, since the definition of $\mid$ does not allow for $n = 0$ when saying $n \mid d$.

1 $\frac{1}{2}$

[MULTIPLE CHOICE] Which of the following statements are true ? **SCORE: ___ / 3 POINTS**

(1) The set of prime numbers is closed under addition.
(2) The set of composite numbers is closed under addition.
(3) The set of composite numbers is closed under multiplication.

[a]	none of the above	[b]	(1) only	[c]	(2) only
[d]	(3) only 3	[e]	(1) & (3) only	[f]	(2) & (3) only

Write a formal proof of the following statement. **SCORE: ___ / 8 POINTS**

Do NOT use any properties regarding the sum, difference or product of even/odd integers mentioned in section 4.2 of your textbook.

For all even integers m, and all odd integers n, $5m^2 - n^2$ is odd.

PROOF: Let m be a particular but arbitrarily chosen even integer, $\frac{1}{2}$ and let n be a particular but arbitrarily chosen odd integer. $\frac{1}{2}$

$\frac{1}{2}$ By the definition of even, $m = 2a$ for some integer a. 1

$\frac{1}{2}$ By the definition of odd, $n = 2b + 1$ for some integer b. 1

$5m^2 - n^2 = 20a^2 - 4b^2 - 4b - 1 = 2(10a^2 - 2b^2 - 2b - 1) + 1$ 2

$\frac{1}{2}$ and $10a^2 - 2b^2 - 2b - 1$ is an integer since Z is closed under $\times$ and $-$. $\frac{1}{2}$

By the definition of odd, $5m^2 - n^2$ is odd.

$\frac{1}{2}$ $\frac{1}{2}$