SCORE: ___ / 30 POINTS

Suppose you wanted to prove the following statement:

SCORE: ___/3 POINTS

If the sum of two even integers is perfect, then the integers are amicable.

Write only the first sentence of a formal proof.

NOTE: "Amicable" and "perfect" are actual properties of numbers. You do not need to know what they mean to answer this question.

Let x and y be two particular but arbitrarily chosen even integers such that x + y is perfect.







Write a formal proof of the following statement.

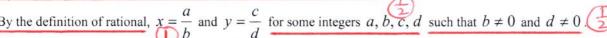
SCORE: /8 POINTS

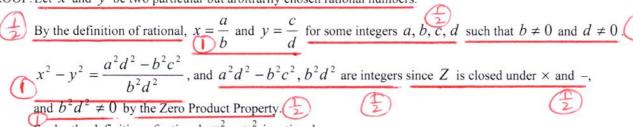
Do NOT use any properties regarding the closure of the set of rational numbers.

The difference of the squares of two rational numbers is rational.

PROOF: Let x and y be two particular but arbitrarily chosen rational numbers.











So, by the definition of rational, $x^2 - y^2$ is rational.



Prove that the following statement is false.

SCORE: ___/3 POINTS

For all integers m, if m is prime, then $m^2 - 2$ is prime.

COUNTEREXAMPLE: $\underline{m=11} \rightarrow \underline{m^2-2=119} = \underline{7 \times 17}$







An integer n is prime if and only if n > 1 and for all positive integers r and s, if n = rs, then r = 1 or s = 1







MUST BE"IF/THEN"

Is the following statement true or false? Explain your answer briefly. (You do not need to write a formal proof.)

SCORE: ___ / 2 POINTS

$$\forall x \in Z, \quad x \mid 2x$$

The statement is fall

COUNTEREXAMPLE: x = 0, since the definition of | does not allow for n = 0 when saying $n \mid d$.



[MULTIPLE CHOICE] Which of the following statements are true?

SCORE: ___ / 3 POINTS

- (1) The set of prime numbers is closed under addition.
- (2)The set of composite numbers is closed under addition.
- The set of composite numbers is closed under multiplication. (3)

[a] none of the above

[b] (1) only [c] (2) only

[d] (3) only

[e] (1) & (3) only [f](2) & (3) only

Write a formal proof of the following statement.

SCORE: ___/8 POINTS

Do NOT use any properties regarding the sum, difference or product of even/odd integers mentioned in section 4.2 of your textbook.

For all even integers m, and all odd integers n, $5m^2 - n^2$ is odd.

PROOF: Let m be a particular but arbitrarily chosen even integer, and let n be a particular but arbitrarily chosen odd integer.

By the definition of even, m = 2a for some integer a

By the definition of odd, n = 2b + 1 for some integer b

$$5m^2 - n^2 = 20a^2 - 4b^2 - 4b - 1 = 2(10a^2 - 2b^2 - 2b - 1) + 1$$

and $10a^2 - 2b^2 - 2b - 1$ is an integer since Z is closed under \times and -.

By the definition of odd, $5m^2 - n^2$ is odd.

