Ma	th 2	24		
Qu	iz 3	Ver	sion	B
	Feb			

NAMES YOU ASKED TO BE CALLED IN CLASS:

SCORE: ___ / 30 POINTS

One of the statements below is true, and the other is false.

SCORE: ___ / 6 POINTS

Prove the statement that is true, and disprove the statement that is false (ie. show that the false statement is false).

- [a] For all positive integers a, b, and c, if $ab \mid c$, then $a \mid c$ and $b \mid c$.
- [b] For all positive integers a, b, and c, if $a \mid bc$, then $a \mid b$ or $a \mid c$.

[a] is true.

Proof: Let a,b, and c be particular but arbitrarily chosen positive integers such that $ab \mid c$ By definition of \mid , c = abk for some $k \in Z$.

So, c = a(bk) = b(ak) where bk, $ak \in Z$ by closure of Z under \times .

Therefore, by the definition of \mid , $a \mid c$ and $b \mid c$.

[b] is false.

Counterexample: a=4, b=2 and c=2 $4 \mid (2 \times 2) \text{ since } 4 = (2 \times 2) \times 1$ but $4 \mid 2$.

Counterexample: a=4, b=2 and c=2 $4 \mid (2 \times 2) \times 1$ $5 \mid$

Prove the following statement.

SCORE: ___ / 8 POINTS

For all positive integers c, if $3 \not/ c$, then $(c^2 - 3c) \mod 3 = 1$.

NOTE: Do NOT use any properties of "mod" unless they were proven in this class or you prove them here (NOT recommended).

Proof: Let c be a particular but arbitrarily chosen positive integer such that $3 \nmid c$.

By definition of $|, c \neq 3q \text{ for any } q \in Z$

By the quotient remainder theorem, c = 3q + 1 or c = 3q + 2 for some $q \in Z$

CASE 1: c = 3q + 1

$$c^2 - 3c = (3q + 1)^2 - 3(3q + 1) = 9q^2 + 6q + 1 - 9q - 3 = 9q^2 - 3q - 2 = 3(3q^2 - q - 1) + 1$$
where $3q^2 - q - 1 \in Z$ by closure of Z under \times and $+$.

By definition of mod, $(c^2 - 3c) \mod 3 = 1$.

CASE 2: c = 3q + 2

$$c^2 - 3c = (3q + 2)^2 - 3(3q + 2) = 9q^2 + 12q + 4 - 9q - 6 = 9q^2 + 3q - 2 = 3(3q^2 + q - 1) + 1$$
where $3q^2 + q - 1 \in Z$ by closure of Z under \times and $+$.

By definition of mod, $(c^2 - 3c) \mod 3 = 1$.

Therefore, $c^2 \mod 3 = 1$.

One of the statements below is true, and the other is false.

SCORE: ___ / 8 POINTS

Prove the statement that is true, and disprove the statement that is false (ie. show that the false statement is false).

NOTE: Do NOT use any properties of "mod" unless they were proven in this class or you prove them here (NOT recommended).

- $\forall n \in \mathbb{Z}^+, n \mod 3 = 2 \rightarrow n^2 \mod 3^2 = 2^2$ [a]
- For all integers a and b, if $a \mod 8 = 7$ and $b \mod 12 = 5$, then $ab \mod 4 = 3$. [b]

HINT: The shortest, simplest proof for the true statement is a direct proof from definitions.

[a] is false.

Counterexample: n = 5

$$5 \mod 3 = 2 \text{ since } 5 = 3(1) + 2$$

but
$$5^2 \mod 3^2 = 25 \mod 9 = 7$$
 since $25 = 9(2) + 7$

 $5 \mod 3 = 2 \text{ since } 5 = 3(1) + 2$ (DON'T FORGET TO but $5^2 \mod 3^2 = 25 \mod 9 = 7 \text{ since } 25 = 9(2) + 7$

[b] is true.

Proof: Let a and b be particular but arbitrarily chosen integers such that $a \mod 8 = 7$ and $b \mod 12 = 5$.

By definition of mod,
$$a = 8m + 7$$
 for some $m \in \mathbb{Z}$.

By definition of mod,
$$b = 12n + 5$$
 for some $n \in \mathbb{Z}$.

$$ab = (8m + 7)(12n + 5) = 96mn + 40m + 84n + 35 = 4(24mn + 10m + 21n + 8) + 3$$

where
$$24mn + 10m + 21n + 8 \in \mathbb{Z}$$
 by closure of \mathbb{Z} under \times and $+$.

Therefore, by definition of mod,
$$ab \mod 4 = 3$$
.

Prove the following statement.

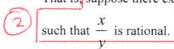
SCORE: ___ / 8 POINTS

The quotient of a non-zero rational number divided by an irrational number is irrational.

Proof by contradiction:

Suppose not.

That is, suppose there exist a non-zero rational number x and an irrational number y



By the definition of rational, $x = \frac{a}{b}$ and $\frac{x}{y} = \frac{c}{d}$ for some $a, b, c, d \in \mathbb{Z}$ where $b, d \neq 0$.

So, xd = cv.

Since $x, d \neq 0$, by zero product property, $xd \neq 0$, and by zero product property, $c \neq 0$.

$$y = \frac{x}{\frac{x}{y}} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$
 where $ad, bc \in Z$ by closure of Z under \times ,

and $bc \neq 0$ by zero product property

So, by definition of rational, y is rational.

But, y is rational and y is irrational. (CONTRADICTION)

So, by contradiction, the quotient of a non-zero rational number divided by an irrational number is irrational.