Math 22
Quiz 6
Fri Feb 24, 2012

NAMES YOU ASKED TO BE CALLED IN CLASS:

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SCORE: ___ / 30 POINTS

Let $F_0, F_1, F_2, \ldots$ be the Fibonacci sequence. Prove that $F_n = 2F_{n-1} - F_{n-3}$ for all integers $n \geq 3$. **SCORE: ___ / 4 PTS**

NOTE: You do NOT need to write a formal proof. Simply show the algebra.

$$F_{n-1} = F_{n-2} + F_{n-3} \longrightarrow F_{n-2} = F_{n-1} - F_{n-3}$$

$$F_n = F_{n-1} + F_{n-2} \longrightarrow F_n = F_{n-1} + F_{n-1} - F_{n-3}$$

$$= 2F_{n-1} - F_{n-3}$$

Let $A = \{a, c, f\}$ and $B = \{a, b, e\}$ be subsets of the universal set $\{a, b, c, d, e, f, g\}$. **SCORE: ___ / 4 PTS**

[a] Find $B \cup A^C$.

$A^c = \{b, d, e, g\}$ 1

$B \cup A^c = \{a, b, d, e, g\}$ 1

[b] Find $B - (B - A)$.

$B - A = \{b, e\}$ 1

$B - (B - A) = \{a\}$ 1

Let $A = \{x \in Z \mid x = 6k - 4$ for some $k \in Z\}$ and $B = \{y \in Z \mid y = 3h + 2$ for some $h \in Z\}$. **SCORE: ___ / 7 PTS**

Prove that $A \subseteq B$.

PROOF: LET $x \in A$ 1

SO, $x \in \mathbb{Z}$ AND $x = 6k - 4$ FOR SOME $k \in \mathbb{Z}$ 1

SO, $x = 3(2k-2)+2$ 1 WHERE $2k-2 \in \mathbb{Z}$ BY CLOSURE OF $\mathbb{Z}$ UNDER $\times$ AND $-$. 1

AND $x \in \mathbb{Z}$ $\frac{1}{2}$

SO, $x \in B$ 1

SO, $A \subseteq B$ 1 BY DEF'N OF $\subseteq$ $\frac{1}{2}$

Fill in the blanks by writing the **symbolic** translations. SCORE: ___ / 2 PTS

Do not use $\cup$, $\cap$, $-$ or ^{C} in your answers.

[a] $A \subseteq B^C$ if and only if $\forall x \in A,\ x \notin B$ 1 .

[b] $x \notin B - A$ if and only if $\sim(x \in B \text{ AND } x \notin A)$ IE. $x \notin B$ OR $x \in A$ 1 .

Let $a_1, a_2, a_3, \ldots$ be a sequence such that SCORE: ___ / 13 PTS

$$a_1 = 1, \quad a_2 = 2 \quad a_3 = 4$$

and $a_n = a_{n-1} + a_{n-2} + 2a_{n-3}$ for all integers $n \geq 4$.

[a] Find the values of a_4, a_5 and a_6. **Verify the values with your partner before continuing to parts [b] and [c].**

$a_4 = a_3 + a_2 + 2a_1 = 4 + 2 + 2 \cdot 1 = 8$ $\frac{1}{2}$

$a_5 = a_4 + a_3 + 2a_2 = 8 + 4 + 2 \cdot 2 = 16$ $\frac{1}{2}$

$a_6 = a_5 + a_4 + 2a_3 = 16 + 8 + 2 \cdot 4 = 32$ $\frac{1}{2}$

[b] Based on the first 6 values of the sequence, guess a general formula for a_n.

$a_n = 2^{n-1}$ 1

[c] Using strong induction, prove that your formula is correct for all integers $n \geq 1$.

BASIS STEP: $n = 1$ $\quad a_1 = 1 = 2^{1-1}$ $\frac{1}{2}$

$n = 2$ $\quad a_2 = 2 = 2^{2-1}$ $\frac{1}{2}$

$n = 3$ $\quad a_3 = 4 = 2^{3-1}$ $\frac{1}{2}$

INDUCTIVE STEP: 1 ASSUME $a_n = 2^{n-1}$ FOR $n = 1, 2, \ldots, k$ FOR SOME $k \in \mathbb{Z}^+$ 1 WHERE $k \geq 3$ 1

1 $a_{k+1} = a_k + a_{k-1} + 2a_{k-2}$ SINCE $k+1 \geq 4$ 1

1 $= 2^{k-1} + 2^{k-2} + 2 \cdot 2^{k-3}$ SINCE $k \geq 1$, $k-1 \geq 1$ AND $k-2 \geq 1$ 1

1 $= \frac{1}{2} \cdot 2^k + \frac{1}{4} \cdot 2^k + \frac{2}{8} \cdot 2^k$

$= 2^k$ $\frac{1}{2}$

SO, BY S.I., $a_n = 2^{n-1}$ FOR ALL $n \in \mathbb{Z}^+$ $\frac{1}{2}$