

Due Mon Mar 19, 2012 @ 10:30AM

NO LATE QUIZZES ACCEPTED

You are strongly encouraged to send a scan of your solution to lobert@deanza.edu no later than Sun Mar 18

You do NOT need to write formal proofs unless specifically instructed to

SCORE: ____ / 30 POINTS

Let $A = \mathbb{Z}^+ - \{1\}$. Define $f : A \rightarrow \mathbb{Z}^+$ by the rule

SCORE: ____ / 7 PTS

$f(x)$ = the greatest factor of x that is not x itself

eg. $f(75) = 25$, since 25 is the greatest factor of 75 other than 75

[a] Is f one-to-one? Explain briefly.

NO. $f(2) = f(3) = 1$

[b] Is f onto? Explain briefly.

YES. IF $y \in \mathbb{Z}^+$, THEN $f(2y) = y$ AND $2y \in \mathbb{Z}^+$ AND $2y \neq 1$, SO $2y \in A$
[THE TWO SMALLEST POSITIVE FACTORS OF $2y$ ARE 1 AND 2, SO

[c] Find $f^{-1}(\{1, 5\})$. Write your answer in proper notation. Do NOT use ellipsis (...) notation.

$\{x \in \mathbb{Z}^+ \mid x \text{ IS PRIME OR } x = 10 \text{ OR } x = 15 \text{ OR } x = 25\}$
THE ONLY FACTOR OF A PRIME WHICH IS NOT THAT PRIME IS 1. 5 IS THE GREATEST FACTOR ONLY IF THE "OTHER" FACTOR IS LESS THAN OR EQUAL TO 5 AND PRIME
THE TWO GREATEST ARE y AND $2y$

[d] If $g(x)$ = the greatest factor of x that is not x itself, and g has domain \mathbb{Z}^+ and co-domain \mathbb{Z}^+ , is g well-defined? Explain briefly.

NO. $g(1) = -1 \notin \mathbb{Z}^+$

Let R be a relation on $\wp(\mathbb{Z})$ defined by

SCORE: ____ / 4 PTS

XRY if and only if $X \cap Y = Y$

NOTE: Do NOT use an alternate definition of R .

NOTE: You may use set algebra (Theorem 6.2.2) instead of writing formal proofs below. State all laws used as justification.

[a] Is R transitive? If yes, write a proof. If no, give a counterexample and show how it indicates R is not transitive.

YES. $XRY \wedge YRZ \rightarrow X \cap Y = Y \wedge Y \cap Z = Z$
 $\rightarrow X \cap Z = X \cap (Y \cap Z)$
 $= (X \cap Y) \cap Z$ ASSOCIATIVE
 $= Y \cap Z$
 $= Z \rightarrow XRZ$

[b] Is R symmetric? If yes, write a proof. If no, give a counterexample and show how it indicates R is not symmetric.

NO. $\{1\} \cap \emptyset = \emptyset \rightarrow \{1\} R \emptyset$
 $\emptyset \cap \{1\} \neq \{1\} \rightarrow \emptyset \not R \{1\}$

Let R be the equivalence relation on $A = \{1, 2, 4, 5, 6, 8\}$ defined by

SCORE: ___ / 3 PTS

$$xRy \text{ if and only if } 5 \mid (2x^2 + 3y^2)$$

Find the partition induced on A by R . You do not need to prove that R is an equivalence relation.

$$\{1, 4, 6\} \cup \{2, 8\} \cup \{5\}$$

Let R be an equivalence relation on set A . WRITE A FORMAL PROOF for the following statement.

SCORE: ___ / 5 PTS

Use the definitions in sections 8.2 and 8.3 but do NOT use any of the lemmas, theorems or homework exercises as justification.

For all $a, b, c, d \in A$, if $a \in [b]$ and $c \in [d]$ and $c \in [b]$, then $d \in [a]$.

LET $a, b, c, d \in A$ SUCH THAT $a \in [b]$, $c \in [d]$ AND $c \in [b]$.

SO, aRb , cRd AND cRb BY DEF'N OF $[]$.

SO, bRa AND dRc BY SYMMETRY

SO, dRb BY TRANSITIVITY (SINCE dRc AND cRb).

SO, dRa BY TRANSITIVITY (SINCE dRb AND bRa)

SO, $d \in [a]$ BY DEF'N OF $[]$

Let $A = \{x \in \mathbb{R} \mid 0 < x < 1\}$ and $B = \{x \in \mathbb{R} \mid a < x < b\}$ where $a, b \in \mathbb{R}$ and $a < b$.

SCORE: ___ / 6 PTS

Prove that A and B have the same cardinality by finding a one-to-one correspondence $f: A \rightarrow B$.

You must WRITE A FORMAL PROOF that your function f is one-to-one and onto. You do NOT need to prove f is a function.

LET $f: A \rightarrow B$ BE DEFINED BY $f(x) = (b-a)x + a$

LET $x, y \in A$ SUCH THAT $f(x) = f(y)$

$$\text{SO, } (b-a)x + a = (b-a)y + a$$

$$\text{SO, } (b-a)x - (b-a)y = 0$$

$$\text{SO, } (b-a)(x-y) = 0$$

$$\text{SINCE } b > a, b-a \neq 0$$

$$\text{SO, } x-y = 0$$

$$\text{SO, } x=y \quad \text{SO, } f \text{ is 1-1}$$

LET $y \in B$

$$\text{LET } x = \frac{y-a}{b-a}$$

$$\text{SINCE } a < y < b$$

$$\text{SO, } 0 < y-a < b-a$$

$$\text{AND } 0 < \frac{y-a}{b-a} < 1$$

$$\text{SO, } x \in A$$

$$\text{AND } f(x) = (b-a)\frac{y-a}{b-a} + a = y \quad \text{SO, } f \text{ IS ONTO}$$

How many 7 digit code numbers (which may start with 0's) contain two different digits that are repeated twice each, and one additional digit that is repeated three times? (eg. 0711701)

SCORE: ___ / 5 PTS

Show the reasoning process that gives your answer. (ie. Do NOT write a program to generate and count all the codes.)

See the combinatorics handout on my website for ideas.

1) PICK 2 NUMBERS TO BE REPEATED TWICE $C(10, 2)$

2) PICK 2 POSITIONS FOR THE SMALLER NUMBER $C(7, 2)$

3) PICK 2 POSITIONS FOR THE LARGER NUMBER $C(5, 2)$

4) PICK A NUMBER FOR ALL REMAINING POSITIONS $C(8, 1)$

$$C(10, 2) * C(7, 2) * C(5, 2) * C(8, 1)$$