Fill in the blanks. **SCORE: _____ / 8 PTS**

[a] The values $a_1, a_2, a_3, a_4, \cdots$ are called the TERMS of a sequence.

[b] If you are given the first term of a sequence, and all other terms of the sequence are defined using previous terms, then the sequence is said to be defined RECURSIVELY.

[c] A sequence is called a/an GEOMETRIC sequence if the ratios between consecutive terms are the same.

[d] For the sum $\sum_{t=3}^{8} \frac{2t-5}{t!}$, 8 is called the UPPER LIMIT OF SUMMATION

Using mathematical induction, prove that 3 is a factor of $4^n - 1$ for all positive integers n. **SCORE: _____ / 24 PTS**

BASIS STEP: $n=1$

$4^1 - 1 = 3$

3 IS A FACTOR OF $4^1 - 1$.

INDUCTIVE STEP: ASSUME 3 IS A FACTOR OF $4^k - 1$ FOR SOME INTEGER $k \geq 1$.

$4^{k+1} - 1 = 4 \cdot 4^k - 1$

$= 4 \cdot 4^k - 4 + 4 - 1$

$= 4(4^k - 1) + 3$

THIS WAS EXAMPLE 5 ON PAGE 675 OF YOUR TEXTBOOK

3 IS A FACTOR OF $4^k - 1$,

SO 3 IS A FACTOR OF $4(4^k - 1)$

AND 3 IS A FACTOR OF 3

SO 3 IS A FACTOR OF $4(4^k - 1) + 3 = 4^{k+1} - 1$

BY M.I., 3 IS A FACTOR OF $4^n - 1$ FOR ALL INTEGERS $n \geq 1$

OR

$4^{k+1} - 1 = 4^{k+1} - 4^k + 4^k - 1$

$= 4 \cdot 4^k - 4^k + (4^k - 1)$

$= 3 \cdot 4^k + (4^k - 1)$

3 IS A FACTOR OF BOTH $3 \cdot 4^k$ AND $4^k - 1$,

SO 3 IS A FACTOR OF $3 \cdot 4^k + (4^k - 1) = 4^{k+1} - 1$

Calculate $\binom{12}{3}$.

SCORE: _____ / 8 PTS

$$\frac{12!}{9!\,3!} = \frac{\overset{2}{12}\cdot 11\cdot 10\cdot 9!}{9!\cdot 6} = 220$$

Write the repeating decimal $0.6\overline{27}$ as a fraction (integer divided by integer) by using a series formula.

SCORE: _____ / 20 PTS

NOTE: Only the " 27 " is repeated.

$$0.6\overline{27} = 0.6 + 0.027 + 0.00027 + 0.0000027 + \dots$$

$$= \frac{3}{5} + \frac{27}{1000} + \frac{27}{100000} + \frac{27}{10000000} + \dots$$

$$= \frac{3}{5} + \frac{\frac{27}{1000}}{1 - \frac{1}{100}}$$

$$= \frac{3}{5} + \frac{\overset{3}{27}}{1000}\cdot\frac{100}{\underset{11}{99}}$$

$$= \frac{3}{5} + \frac{3}{110}$$

$$= \frac{66+3}{110}$$

$$= \frac{69}{110}$$

Write the following series using sigma notation.

SCORE: _____ / 20 PTS

$$\frac{47}{3} - \frac{43}{6} + \frac{39}{12} - \frac{35}{24} + \dots - \frac{11}{1536}$$

← ARITHMETIC $d = -4$

← GEOMETRIC $r = 2$

$$\sum_{n=1}^{10} (-1)^{n-1} \frac{47-4(n-1)}{3\cdot 2^{n-1}}$$

$47 - 4(n-1) = 11$

$-4(n-1) = -36$

$n - 1 = 9$

$n = 10$

OR

$$\sum_{n=0}^{9} (-1)^n \frac{47-4n}{3\cdot 2^n}$$

Use the binomial theorem to find the coefficient a of the ax^2y^9 term in the expansion of $(3x+2y)^{11}$. **SCORE: _____ / 15 PTS**

NOTE: You do NOT need to find the complete expansion of the binomial.
NOTE: Do NOT use Pascal's Triangle to find the needed binomial coefficient.

$r=9$

$${}_{11}C_9 (3x)^2 (2y)^9 = \frac{11!}{2!\,9!}(9x^2)(512y^9)$$

$$= \frac{11 \cdot \cancel{10}^{5} \cdot 9!}{\cancel{2} \cdot 9!} 4608x^2y^9$$

$$= 253{,}440x^2y^9$$

The first row of a stadium has 85 seats, and the second row has 89 seats. If the number of seats increases by the same number from each row to the next, and the last row has 173 seats, how many seats are in the entire stadium ? **SCORE: _____ / 20 PTS**

ARITHMETIC $d = 89 - 85 = 4$

$$85 + 4(n-1) = 173$$
$$4(n-1) = 88$$
$$n-1 = 22$$
$$n = 23$$

$$S = \frac{23}{2}(85+173) = 2967$$

If the first term of a geometric series is $8x^9$, and the second term is $24x^3$, find the ninth term. **SCORE: _____ / 15 PTS**

$$r = \frac{24x^3}{8x^9} = \frac{3}{x^6}$$

$$a_9 = 8x^9\left(\frac{3}{x^6}\right)^8 = 8x^9\left(\frac{6561}{x^{48}}\right) = \frac{52488}{x^{39}}$$

[redacted]

Use the binomial theorem to completely expand $(2x^2 - 5y)^5$.

You may use Pascal's Triangle to find the needed binomial coefficients.

SCORE: _____ / 20 PTS

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$(2x^2)^5 + 5(2x^2)^4(-5y) + 10(2x^2)^3(-5y)^2$

$+ 10(2x^2)^2(-5y)^3 + 5(2x^2)(-5y)^4 + (-5y)^5$

$= 32x^{10} - 400x^8y + 2000x^6y^2 - 5000x^4y^3 + 6250x^2y^4 - 3125y^5$