Fill in the blanks. **SCORE: _____ / 8 PTS**

[a] The values $a_1, a_2, a_3, a_4, \cdots$ are called the TERMS of a sequence.

[b] If you are given the first term of a sequence, and all other terms of the sequence are defined using previous terms, then the sequence is said to be defined RECURSIVELY.

[c] A sequence is called a/an ARITHMETIC sequence if the differences between consecutive terms are the same.

[d] For the sum $\sum_{t=8}^{11} \frac{2t-5}{t!}$, 8 is called the LOWER LIMIT OF SUMMATION

Using mathematical induction, prove that $n \leq 2^n$ for all positive integers n. **SCORE: _____ / 24 PTS**

BASIS STEP: $n=1$

$1 \leq 2^1 = 2$

INDUCTIVE STEP: ASSUME $k \leq 2^k$ FOR SOME INTEGER $k \geq 1$

$2^{k+1} = 2 \cdot 2^k \geq 2k = (k+1)+(k-1) \geq k+1$

[SINCE $k \geq 1$

$k-1 \geq 0$]

BY M.I., $n \leq 2^n$ FOR ALL INTEGERS $n \geq 1$

THIS WAS EXAMPLE 4 ON PAGE 675 OF YOUR TEXTBOOK

OR

$k+1 \leq 2^k+1 \leq 2^k+2^k = 2 \cdot 2^k = 2^{k+1}$

[SINCE $k \geq 1$

$2^k \geq 2 > 1$]

BY M.I., $n \leq 2^n$ FOR ALL INTEGERS $n \geq 1$

Calculate $\binom{9}{3}$. **SCORE: _____ / 8 PTS**

$$\frac{9!}{6!\,3!} = \frac{\overset{3}{9}\cdot \overset{4}{8}\cdot 7\cdot 6!}{6!\cdot \cancel{6}_{2}} = 84$$

Write the repeating decimal $0.4\overline{36}$ as a fraction (integer divided by integer) by using a series formula. **SCORE: _____ / 20 PTS**

NOTE: Only the "36" is repeated.

$$0.4\overline{36} = 0.4 + 0.036 + 0.00036 + 0.0000036 + \cdots$$

$$= \frac{2}{5} + \frac{36}{1000} + \frac{36}{100000} + \frac{36}{10000000} + \cdots$$

$$= \frac{2}{5} + \frac{\frac{36}{1000}}{1 - \frac{1}{100}}$$

$$= \frac{2}{5} + \frac{\overset{2}{\cancel{4}}\cancel{36}}{\underset{5}{\cancel{1000}}} \cdot \frac{\cancel{100}}{\underset{11}{\cancel{99}}}$$

$$= \frac{2}{5} + \frac{2}{55}$$

$$= \frac{22+2}{55}$$

$$= \frac{24}{55}$$

Write the following series using sigma notation. **SCORE: _____ / 20 PTS**

$$\frac{57}{2} - \frac{51}{6} + \frac{45}{18} - \frac{39}{54} + \cdots - \frac{3}{39366}$$

← ARITHMETIC $d=-6$

← GEOMETRIC $r=3$

$$\sum_{n=1}^{10} (-1)^{n-1} \frac{57-6(n-1)}{2\cdot 3^{n-1}}$$

$$57-6(n-1)=3$$
$$-6(n-1)=-54$$
$$n-1=9$$
$$n=10$$

OR

$$\sum_{n=0}^{9} (-1)^{n} \frac{57-6n}{2\cdot 3^{n}}$$

Use the binomial theorem to find the coefficient a of the ax^2y^9 term in the expansion of $(2x+3y)^{11}$. **SCORE: _____ / 15 PTS**

NOTE: You do NOT need to find the complete expansion of the binomial.

NOTE: Do NOT use Pascal's Triangle to find the needed binomial coefficient.

$r=9$

$${}_{11}C_9(2x)^2(3y)^9 = \frac{11!}{2!9!}(4x^2)(19683y^9)$$

$$= \frac{11\cdot \overset{5}{\cancel{10}}\cdot \cancel{9!}}{\cancel{2}\cdot\cancel{9!}}\ 78732x^2y^9$$

$$= 4{,}330{,}260x^2y^9$$

The first row of a stadium has 85 seats, and the second row has 88 seats. If the number of seats increases by the same number from each row to the next, and the last row has 154 seats, how many seats are in the entire stadium ? **SCORE: _____ / 20 PTS**

ARITHMETIC $d=88-85$

$$85+3(n-1)=154$$

$$3(n-1)=69$$

$$n-1=23$$

$$n=24$$

$$S=\frac{24}{2}(85+154)=2868$$

If the first term of a geometric series is $6x^6$, and the second term is $24x^2$, find the ninth term. **SCORE: _____ / 15 PTS**

$$r=\frac{24x^2}{6x^6}=\frac{4}{x^4}$$

$$a_9=6x^6\left(\frac{4}{x^4}\right)^8=6x^6\left(\frac{65536}{x^{32}}\right)=\frac{393216}{x^{26}}$$

Use the binomial theorem to completely expand $(5x^2 - 2y)^5$.
You may use Pascal's Triangle to find the needed binomial coefficients.

SCORE: _____ / 20 PTS

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$$(5x^2)^5 + 5(5x^2)^4(-2y) + 10(5x^2)^3(-2y)^2$$
$$+ 10(5x^2)^2(-2y)^3 + 5(5x^2)(-2y)^4 + (-2y)^5$$
$$= 3125x^{10} - 6250x^8y + 5000x^6y^2 - 2000x^4y^3 + 400x^2y^4 - 32y^5$$