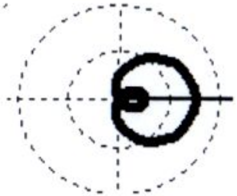
[redacted]

Fill in the blanks. **SCORE: _____ / 15 PTS**



[] The graph of $r = \sin 8\theta$ is a rose curve with ___16___ petals.

[] The shape of the graph of $r^2 = \sin 2\theta$ is called a/an ___LEMNISCATE___.

[] The shape of the graph of $r = \frac{1}{2} + \cos\theta$ shown on the upper right is called a/an ___LIMAÇON___.

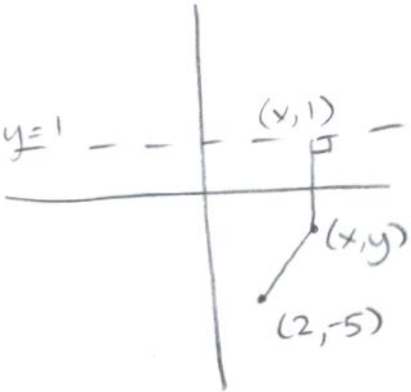
[] For a parabola, the ___AXIS OF SYMMETRY___ passes through the vertex and the ___FOCUS___,

and is perpendicular to the ___DIRECTRIX___.

[redacted]

Using the distance-based definition of a parabola, find the standard form of the equation of the parabola containing all points whose distance to the point $(2, -5)$ equals their distance to the line $y = 1$.

SCORE: _____ / 15 PTS



$$\sqrt{(x-2)^2+(y+5)^2} = 1-y$$

$$(x-2)^2+(y+5)^2 = (1-y)^2$$

$$(x-2)^2 + y^2+10y+25 = 1-2y+y^2$$

$$(x-2)^2 = -12y-24$$

$$(x-2)^2 = -12(y+2)$$

Consider the graph of the polar equation $r = {}^{-}6 + 2\sin\theta$. **SCORE: ____ / 8 PTS**

[a] How far from the pole is the farthest point on the graph ?

$-8 \le r \le -4$

8 UNITS

[b] Find all values of θ (for $0 \le \theta < 2\pi$) where the graph passes through the pole.

$0 = -6 + 2\sin\theta$

$\sin\theta = 3$ NO SOL'N $\longrightarrow$ GRAPH NEVER PASSES THROUGH POLE

Write the eccentricity-based definition of a conic given in section 10.9. **SCORE: _____ / 7 PTS**

GIVEN A POINT F AND A LINE D,
A CONIC IS THE LOCUS OF POINTS P
SUCH THAT $\frac{PF}{PQ}$ IS A FIXED CONSTANT (WHERE PQ IS THE PERPENDICULAR DISTANCE FROM P TO D)

Consider the graph of the polar equation $r^3 = \cos 2\theta$.

SCORE: _____ / 15 PTS

[a] Determine whether the graph is symmetric with respect to the pole, the polar axis, and $\theta = \frac{\pi}{2}$.

$(r, -\theta)$

$r^3 = \cos 2(-\theta)$

$r^3 = \cos \theta$

SYM OVER POLAR AXIS

$(r, \pi + \theta)$

$r^3 = \cos 2(\pi + \theta)$

$r^3 = \cos(2\pi + 2\theta)$

$r^3 = \cos 2\pi \cos 2\theta - \sin 2\pi \sin \theta$

$r^3 = \cos 2\theta$

SYM OVER POLE

SO, ALSO SYM OVER $\theta = \frac{\pi}{2}$

[b] What is the minimum interval for θ that you would need to plot points before using symmetry to finish drawing the graph ?

$[0, \frac{\pi}{2}]$

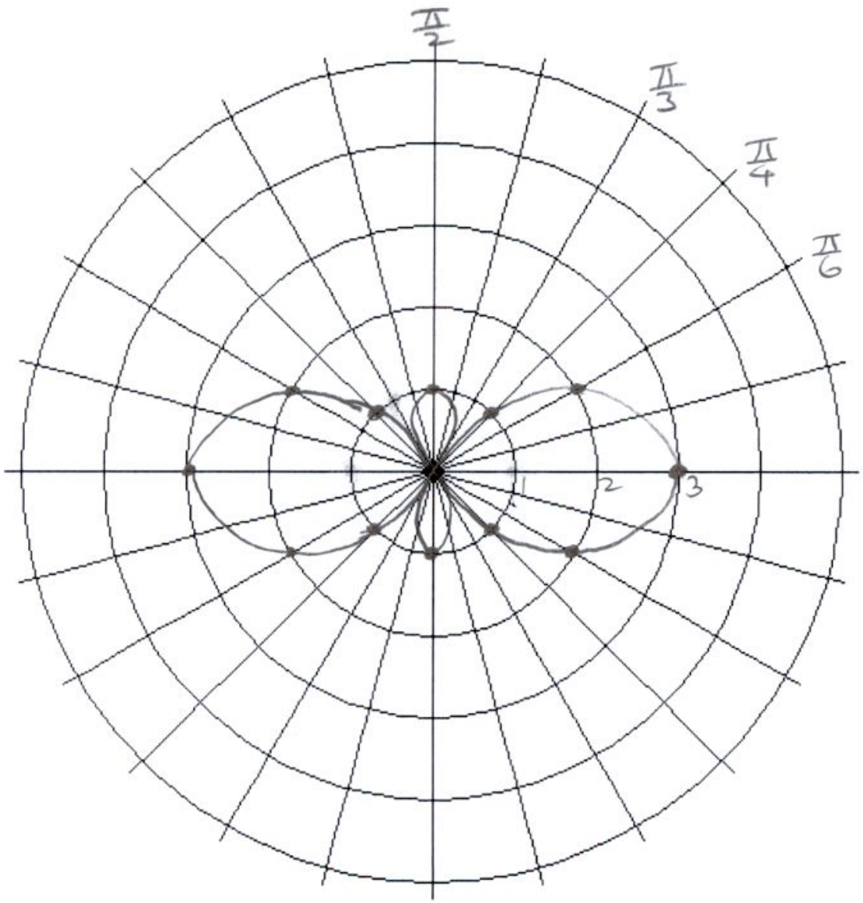
[redacted]

Your quiz partner has done all preliminary analysis on the polar equation $r = 4\cos^2\theta - 1$. Here are the results: **SCORE: _____ / 15 PTS**

The symmetry tests with respect to the pole and the polar axis showed the graph is symmetric in those ways.
The range of $r-$values is $-1 \le r \le 3$.

$4\cos^2\theta - 1 = 0$ when $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$.

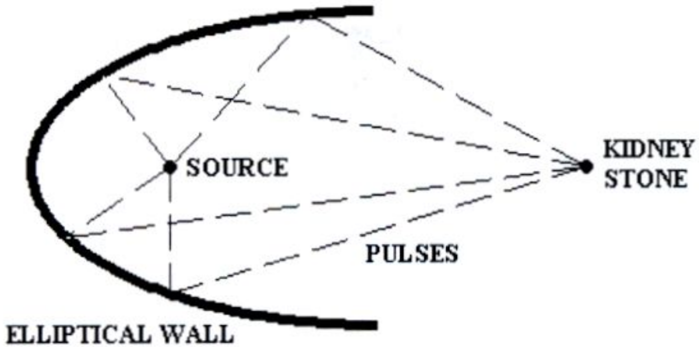
Sketch the graph. Show the polar co-ordinates of the fewest points you need to plot before using symmetry to finish the graph.



θ	r
0	3
$\frac{\pi}{6}$	2
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	0
$\frac{\pi}{2}$	-1

[redacted]

A lithotripter is a medical device for breaking down kidney stones using acoustic pulses. **SCORE: _____ / 15 PTS**
The patient is placed so their kidney stone is at the focus of an ellipse. A source of acoustic pulses is placed at the other focus.
Pulses sent from the source reflect off an elliptical wall and meet again at the other focus, where they shatter the kidney stone.



If the source and the kidney stone are 4 feet apart, and each pulse travels a total of 6 feet from the source to the wall to the kidney stone, find the standard form of the rectangular equation of the ellipse. (Assume that the origin is the midpoint of the source and the kidney stone.)

FOCAL LENGTH = 4

$c = \frac{1}{2}(4) = 2$

MAJOR AXIS = 6

$a = \frac{1}{2}(6) = 3$

$3^2 = 2^2 + b^2$

$b^2 = 5$

$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$

Consider the conic with polar equation $r = \dfrac{15}{3 - 2\sin\theta}$. **SCORE: _____ / 30 PTS**

[a] Is the conic a circle, a parabola, an ellipse or a hyperbola ?

$r = \dfrac{5}{1 - \frac{2}{3}\sin\theta}$ $e = \frac{2}{3} \rightarrow$ ELLIPSE

[b] Find the equation of the directrix.

$ep = 5 \rightarrow \frac{2}{3}p = 5 \rightarrow p = \frac{15}{2}$ $y = -\frac{15}{2}$

[c] Find the rectangular co-ordinates of the center of the conic.

θ	r
0	5
$\pi/2$	15
π	5
$3\pi/2$	3

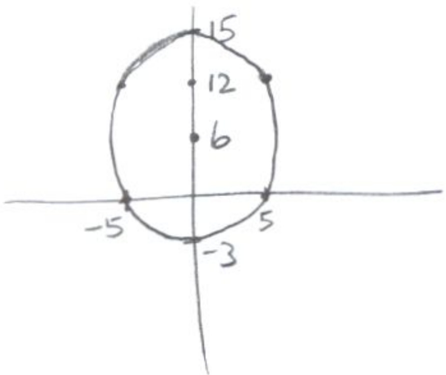
$\left(0, \frac{15-3}{2}\right) = (0, 6)$

[d] Find the rectangular co-ordinates of the endpoints of **BOTH** latera recta. **NOTE: "Latera recta" is the plural of "latus rectum".**

FOCUS $= (0, 2 \times 6) = (0, 12)$

ENDPOINTS $= (5, 0)\ (-5, 0)\ (5, 12)\ (-5, 12)$

[e] Sketch the conic. **You do not need to be precise, but you must label the co-ordinates on the x– and y– axes clearly.**



Consider the conic with equation $79 + 16x + 54y + 4x^2 - 9y^2 = 0$.

SCORE: _____ / 30 PTS

[a] Is the conic a circle, a parabola, an ellipse or a hyperbola ?

$A = 4,\ C = -9 \longrightarrow$ HYPERBOLA

[b] Find the standard form of the equation of the conic.

$$4(x^2+4x) - 9(y^2-6y) = -79$$
$$4(x^2+4x+4) - 9(y^2-6y+9) = -79 + 4\cdot 4 - 9\cdot 9$$
$$4(x+2)^2 - 9(y-3)^2 = -144$$
$$\frac{(y-3)^2}{16} - \frac{(x+2)^2}{36} = 1$$

[c] Find the co-ordinates of the vertex/vertices.

$(-2, 3\pm 4) = (-2, 7)$ AND $(-2, -1)$

[d] Find the eccentricity of the conic.

$$c^2 = 16 + 36 = 52$$
$$c = 2\sqrt{13}$$
$$e = \frac{c}{a} = \frac{2\sqrt{13}}{4} = \frac{\sqrt{13}}{2}$$

[e] If the conic is a parabola, find the equation of the directrix.
If the conic is an ellipse, find the endpoints of its minor axis.
If the conic is a hyperbola, find the equations of its asymptotes.

$$y - 3 = \pm \frac{4}{6}(x+2)$$
$$y - 3 = \pm \frac{2}{3}(x+2)$$