[redacted]

Fill in the blanks. **SCORE: _____ / 12 PTS**

[a] If $\mathbf{r} \cdot \mathbf{r} = 18$, then $\| \mathbf{r} \| =$ $\sqrt{18} = 3\sqrt{2}$.

[b] If $\mathbf{q} \cdot \mathbf{q} = 49$, then $\mathbf{q} \times \mathbf{q} =$ 0 .

[c] If the force applied to an object and the direction the object moves are perpendicular to each other, then the work done is 0 .

[d] The expression $\mathbf{r} \cdot (\mathbf{s} \times \mathbf{t})$ is called the TRIPLE SCALAR PRODUCT of vectors $\mathbf{r}$, $\mathbf{s}$ and $\mathbf{t}$.

Let $\mathbf{g} = -\mathbf{i} + 2\mathbf{k}$ and $\mathbf{h} = 2\mathbf{i} + 3\mathbf{j}$.

SCORE: _____ / 18 PTS

[a] Determine if the angle between $\mathbf{g}$ and $\mathbf{h}$ is obtuse, acute or right.

$\langle -1, 0, 2 \rangle \cdot \langle 2, 3, 0 \rangle = -2 + 0 + 0 = -2 < 0$ OBTUSE

[b] Write $\mathbf{g}$ as the sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{h}}\, \mathbf{g}$.

$$\text{PROJ}_{\vec{h}}\, \vec{g} = \frac{\vec{g} \cdot \vec{h}}{\vec{h} \cdot \vec{h}} \vec{h} = \frac{-2}{13} \langle 2, 3, 0 \rangle = \left\langle \frac{-4}{13}, \frac{-6}{13}, 0 \right\rangle$$

$$\vec{g} - \text{PROJ}_{\vec{h}}\, \vec{g} = \langle -1, 0, 2 \rangle - \left\langle -\frac{4}{13}, -\frac{6}{13}, 0 \right\rangle = \left\langle -\frac{9}{13}, \frac{6}{13}, 2 \right\rangle$$

$$\vec{g} = \left\langle \frac{-4}{13}, \frac{-6}{13}, 0 \right\rangle + \left\langle -\frac{9}{13}, \frac{6}{13}, 2 \right\rangle.$$

If $\mathbf{z} = -7\,\mathbf{j} + 4\,\mathbf{k}$ has terminal point $(1, -3, -2)$, find its initial point. **SCORE: _____ / 7 PTS**

$\langle 0, -7, 4 \rangle = \langle 1-x, -3-y, -2-z \rangle$

$1-x=0 \qquad x=1$

$-3-y=-7 \qquad y=4 \qquad (1,4,-6)$

$-2-z=4 \qquad z=-6$

Determine if the points $(1, -3, -2)$, $(-3, -1, -4)$ and $(7, -6, 1)$ are collinear. **SCORE: _____ / 15 PTS**

P Q R

$$\overrightarrow{PQ} = \langle -3-1, -1--3, -4--2 \rangle = \langle -4, 2, -2 \rangle$$

$$\overrightarrow{QR} = \langle 7--3, -6--1, 1--4 \rangle = \langle 10, -5, 5 \rangle$$

$$\langle -4, 2, -2 \rangle = k\langle 10, -5, 5 \rangle \rightarrow \left.\begin{array}{r} -4 = 10k \\ 2 = -5k \\ -2 = 5k \end{array}\right\} k = -\frac{2}{5}$$

COLLINEAR

Find parametric equations for the ellipse with vertices $(3, 7)$ and $(3, -1)$, and foci $(3, 5)$ and $(3, 1)$. **SCORE: _____ / 15 PTS**

CENTER $= \left(\frac{3+3}{2}, \frac{7+(-1)}{2}\right) = (3, 3)$

SEMI-MAJOR AXIS $= \frac{1}{2}(7 - {-1}) = 4$
(VERTICAL)

SEMI-FOCAL LENGTH $= \frac{1}{2}(5-1) = 2$

SEMI-MINOR AXIS $= \sqrt{4^2 - 2^2} = 2\sqrt{3}$

$x = 3 + 2\sqrt{3}\cos t$
$y = 3 + 4\sin t$

Let D be the point $(-3, -1, 2)$.

SCORE: _____ / 14 PTS

Let ℓ be the line $\dfrac{z+3}{4} = \dfrac{x-5}{2} = \dfrac{6-y}{7}$.

Let P be the plane $y - 4z + 2x - 8 = 0$.

[a] Find the symmetric equations of the line through D and perpendicular to P.

DIRECTION VECTOR $= \langle 2, 1, -4 \rangle$

$$\frac{x+3}{2} = \frac{y+1}{1} = \frac{z-2}{-4} \quad \text{OR} \quad \frac{x+3}{2} = y+1 = \frac{2-z}{4}$$

[b] Find the parametric equations of the line through D and parallel to ℓ.

DIRECTION VECTOR $= \langle 2, -7, 4 \rangle$

$x = -3 + 2t$

$y = -1 - 7t$

$z = 2 + 4t$

[redacted]

Let P be the point $(5, -5)$, and Q be the point $(-4, -3)$. **SCORE: _____ / 18 PTS**

[a] If $\mathbf{w}$ is the vector with terminal point P and initial point Q, find the vector with magnitude 7 in the **<u>opposite</u>** direction as $\mathbf{w}$. Write your final answer as a linear combination of $\mathbf{i}$ and $\mathbf{j}$.

$$\overrightarrow{QP} = \langle 5 - {-4}, -5 - {-3} \rangle = \langle 9, -2 \rangle$$

$$\frac{1}{\|\overrightarrow{QP}\|}\overrightarrow{QP} = \frac{1}{\sqrt{85}}\langle 9, -2 \rangle = \left\langle \frac{9}{\sqrt{85}}, \frac{-2}{\sqrt{85}} \right\rangle$$

$$\vec{w} = -7\left\langle \frac{9}{\sqrt{85}}, \frac{-2}{\sqrt{85}} \right\rangle = \left\langle \frac{-63}{\sqrt{85}}, \frac{14}{\sqrt{85}} \right\rangle$$

[b] If an object is moved from Q to P by a force represented by the vector $\langle 3, -7 \rangle$. Find the work done.

$$\langle 3, -7 \rangle \cdot \langle 9, -2 \rangle = 27 + 14 = 41$$

Consider the sphere with equation $x^2 + y^2 + z^2 - 12x + 6y - 10z + 54 = 0$. **SCORE: _____ / 15 PTS**

[a] Find the center and radius of the sphere.

$x^2 - 12x + y^2 + 6y + z^2 - 10z = -54$

$x^2 - 12x + 36 + y^2 + 6y + 9 + z^2 - 10z + 25 = -54 + 36 + 9 + 25$

$(x-6)^2 + (y+3)^2 + (z-5)^2 = 16$

CENTER $(6, -3, 5)$ RADIUS $= 4$

[b] Find and describe the yz – trace of the sphere.

$(0-6)^2 + (y+3)^2 + (z-5)^2 = 16$

$(y+3)^2 + (z-5)^2 = -20$ NO TRACE

[redacted]

Let A be the point $(-3, -1, 2)$, B be the point $(1, -3, -1)$ and C be the point $(2, -2, 1)$. **SCORE: _____ / 21 PTS**

[a] Find the standard (point-normal) form of the equation of the plane containing A, B and C.

$$\overrightarrow{AB} = \langle 1 - {}^-3, -3 - {}^-1, {}^-1 - 2 \rangle = \langle 4, -2, -3 \rangle$$

$$\overrightarrow{BC} = \langle 2 - 1, {}^-2 - {}^-3, 1 - {}^-1 \rangle = \langle 1, 1, 2 \rangle$$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \left| \begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 4 & -2 & -3 & 4 & -2 \\ 1 & 1 & 2 & 1 & 1 \end{array} \right| = -4\vec{i} - 3\vec{j} + 4\vec{k} - (-2\vec{k} - 3\vec{i} + 8\vec{j})$$

$$= -\vec{i} - 11\vec{j} + 6\vec{k}$$

$$-(x+3) - 11(y+1) + 6(z-2) = 0$$

[b] Find the area of the triangle ABC.

$$\frac{1}{2} \| \langle -1, -11, 6 \rangle \| = \frac{1}{2}\sqrt{(-1)^2 + (-11)^2 + 6^2} = \frac{\sqrt{158}}{2}$$

[redacted]

Find the simplified rectangular equation corresponding to the parametric equations $\begin{array}{l} x = 3\ln 2t \\ y = 4t^6 \end{array}$. SCORE: _____ / 15 PTS

$\frac{x}{3} = \ln 2t$

$e^{\frac{x}{3}} = 2t$

$t = \frac{1}{2}e^{\frac{x}{3}}$

$y = 4\left(\frac{1}{2}e^{\frac{x}{3}}\right)^6$

$y = 4\left(\frac{1}{64}e^{2x}\right)$

$y = \frac{1}{16}e^{2x}$