

SCORE: ____ / 20 POINTS

WHERE INDICATED, YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER TO GET FULL CREDIT.

Find the first 4 terms of the sequence defined recursively by $a_1 = 2$, $a_k = k^2 - a_{k-1}$ (for $k \geq 2$).

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$$a_1 = 2$$

➔ MINUS ½ point if you forgot to write $a_1 = 2$

$$a_2 = 2^2 - a_1 = 4 - 2 = \boxed{2}$$

1 point

$$a_3 = 3^2 - a_2 = 9 - 2 = \boxed{7}$$

1 point

$$a_4 = 4^2 - a_3 = 16 - 7 = \boxed{9}$$

1 point

Simplify the expression $\frac{(2n-4)!}{(2n-2)!}$.

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SHOW YOUR WORK.

$$= \frac{(2n-4) \cdots (3)(2)(1)}{(2n-2)(2n-3)(2n-4) \cdots (3)(2)(1)}$$

1½ points

$$= \frac{1}{(2n-2)(2n-3)}$$

1½ points

OR

$$= \frac{(2n-4)!}{(2n-2)(2n-3)(2n-4)!}$$

1½ points

$$= \frac{1}{(2n-2)(2n-3)}$$

1½ points

Find a general formula for the arithmetic sequence whose first term is 7, and whose fourth term is 14.

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SHOW YOUR WORK.

$$a_4 = a_1 + (4-1)d$$

$$\boxed{14 = 7 + 3d}$$

1 point

$$\boxed{d = \frac{7}{3}}$$

½ point

$$\boxed{a_n = 7 + \frac{7}{3}(n-1)}$$

1½ points

Evaluate $\sum_{m=2}^5 m(m-4)$.

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SHOW YOUR WORK. SIMPLIFY YOUR ANSWER.

$$\begin{aligned}
 &= 2(2-4) + 3(3-4) + 4(4-4) + 5(5-4) \\
 &= 2(-2) + 3(-1) + 4(0) + 5(1) \\
 &= \boxed{-4} \quad \boxed{-3} \quad \boxed{+0} \quad \boxed{+5} \\
 &\quad \text{\textcolor{red}{½ point}} \quad \text{\textcolor{red}{½ point}} \quad \text{\textcolor{red}{½ point}} \quad \text{\textcolor{red}{½ point}} \\
 &= \boxed{-2} \\
 &\quad \text{\textcolor{red}{1 point}}
 \end{aligned}$$

Find the first 5 terms of the sequence defined by $a_n = \frac{1 + (-1)^n}{n!}$.

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SIMPLIFY YOUR ANSWERS.

$$a_1 = \frac{1 + (-1)^1}{1!} = \boxed{0} \quad \text{\textcolor{red}{½ point}}$$

$$a_2 = \frac{1 + (-1)^2}{2!} = \boxed{1} \quad \text{\textcolor{red}{½ point}}$$

$$a_3 = \frac{1 + (-1)^3}{3!} = \boxed{0} \quad \text{\textcolor{red}{½ point}}$$

$$a_4 = \frac{1 + (-1)^4}{4!} = \boxed{\frac{1}{12}} \quad \text{\textcolor{red}{½ point}}$$

$$a_5 = \frac{1 + (-1)^5}{5!} = \boxed{0} \quad \text{\textcolor{red}{½ point}}$$

➔ **PLUS ½ point if you got at least 4 of the terms correct**

Fill in the blanks: For the sum $\sum_{k=2}^m a_k$, m is called the upper limit of summation,

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k is called the index (OR dummy index) of summation, and

2 is called the lower limit of summation.

Use sigma notation to write the sum $\frac{1}{4} + \frac{3}{8} + \frac{7}{16} + \frac{15}{32} + \frac{31}{64}$.

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½ point

$$\sum_{n=1}^{\boxed{5}} \frac{\boxed{2^n - 1}}{\boxed{2^{n+1}}}$$

½ point

½ point for numerator, ½ point for denominator ➔ PLUS ½ point if both correct

½ point

OR

½ point

$$\sum_{n=2}^{\boxed{6}} \frac{\boxed{2^{n-1} - 1}}{\boxed{2^n}}$$

½ point

½ point for numerator, ½ point for denominator ➔ PLUS ½ point if both correct

½ point