

SCORE: ____ / 20 POINTS

WHERE INDICATED, YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER TO GET FULL CREDIT.

Find the first 4 terms of the sequence defined recursively by $a_1 = 1$, $a_k = k^2 - a_{k-1}$ (for $k \geq 2$).

SCORE: ____ / 3 POINTS

$$a_1 = 1$$

➔ MINUS ½ point if you forgot to write $a_1 = 1$

$$a_2 = 2^2 - a_1 = 4 - 1 = \boxed{3}$$

1 point

$$a_3 = 3^2 - a_2 = 9 - 3 = \boxed{6}$$

1 point

$$a_4 = 4^2 - a_3 = 16 - 6 = \boxed{10}$$

1 point

Simplify the expression $\frac{(3n-3)!}{(3n-1)!}$.

SCORE: ____ / 3 POINTS

SHOW YOUR WORK.

$$= \frac{(3n-3) \cdots (3)(2)(1)}{(3n-1)(3n-2)(3n-3) \cdots (3)(2)(1)}$$

1½ points

OR

$$= \frac{(3n-3)!}{(3n-1)(3n-2)(3n-3)!}$$

1½ points

$$= \frac{1}{(3n-1)(3n-2)}$$

1½ points

$$= \frac{1}{(3n-1)(3n-2)}$$

1½ points

Find a general formula for the arithmetic sequence whose first term is 6, and whose fourth term is 11.

SCORE: ____ / 3 POINTS

SHOW YOUR WORK.

$$a_4 = a_1 + (4-1)d$$

$$\boxed{11 = 6 + 3d}$$

1 point

$$\boxed{d = \frac{5}{3}}$$

½ point

$$\boxed{a_n = 6 + \frac{5}{3}(n-1)}$$

1½ points

Evaluate $\sum_{m=2}^5 m(m-3)$.

SCORE: ____ / 3 POINTS

SHOW YOUR WORK. SIMPLIFY YOUR ANSWER.

$$= 2(2-3) + 3(3-3) + 4(4-3) + 5(5-3)$$

$$= 2(-1) + 3(0) + 4(1) + 5(2)$$

$$= \boxed{-2} \quad \boxed{+0} \quad \boxed{+4} \quad \boxed{+10}$$

$\frac{1}{2}$ point $\frac{1}{2}$ point $\frac{1}{2}$ point $\frac{1}{2}$ point

$$= \boxed{12}$$

1 point

Find the first 5 terms of the sequence defined by $a_n = \frac{1 - (-1)^n}{n!}$.

SCORE: ____ / 3 POINTS

SIMPLIFY YOUR ANSWERS.

$$a_1 = \frac{1 - (-1)^1}{1!} = \boxed{2} \quad \frac{1}{2} \text{ point}$$

$$a_2 = \frac{1 - (-1)^2}{2!} = \boxed{0} \quad \frac{1}{2} \text{ point}$$

$$a_3 = \frac{1 - (-1)^3}{3!} = \boxed{\frac{1}{3}} \quad \frac{1}{2} \text{ point}$$

$$a_4 = \frac{1 - (-1)^4}{4!} = \boxed{0} \quad \frac{1}{2} \text{ point}$$

$$a_5 = \frac{1 - (-1)^5}{5!} = \boxed{\frac{1}{60}} \quad \frac{1}{2} \text{ point}$$

➔ PLUS $\frac{1}{2}$ point if you got at least 4 of the terms correct

Fill in the blanks: For the sum $\sum_{m=2}^k a_m$, m is called the index (OR dummy index) of summation,

SCORE: ____ / 2 POINTS

k is called the upper limit of summation, and

2 is called the lower limit of summation.

Use sigma notation to write the sum $\frac{1}{4} + \frac{3}{8} + \frac{7}{16} + \frac{15}{32} + \frac{31}{64}$.

SCORE: ____ / 3 POINTS

$\frac{1}{2}$ point $\boxed{5}$

$\frac{1}{2}$ point $\sum \frac{2^n - 1}{2^{n+1}}$

$\frac{1}{2}$ point $\boxed{n=1}$

$\frac{1}{2}$ point for numerator, $\frac{1}{2}$ point for denominator ➔ PLUS $\frac{1}{2}$ point if both correct

OR

$\frac{1}{2}$ point $\boxed{6}$

$\frac{1}{2}$ point $\sum \frac{2^{n-1} - 1}{2^n}$

$\frac{1}{2}$ point $\boxed{n=2}$

$\frac{1}{2}$ point for numerator, $\frac{1}{2}$ point for denominator ➔ PLUS $\frac{1}{2}$ point if both correct