

SCORE: \_\_\_\_ / 20 POINTS

WHERE INDICATED, YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER TO GET FULL CREDIT.

Find the first 5 terms of the sequence defined  $a_n = \frac{2 + (-1)^n}{n!}$ .

SCORE: \_\_\_\_ / 3 POINTS

**SIMPLIFY YOUR ANSWERS.**

$$a_1 = \frac{2 + (-1)^1}{1!} = \boxed{1} \quad \frac{1}{2} \text{ point}$$

$$a_2 = \frac{2 + (-1)^2}{2!} = \boxed{\frac{3}{2}} \quad \frac{1}{2} \text{ point}$$

$$a_3 = \frac{2 + (-1)^3}{3!} = \boxed{\frac{1}{6}} \quad \frac{1}{2} \text{ point}$$

$$a_4 = \frac{2 + (-1)^4}{4!} = \boxed{\frac{1}{8}} \quad \frac{1}{2} \text{ point}$$

$$a_5 = \frac{2 + (-1)^5}{5!} = \boxed{\frac{1}{120}} \quad \frac{1}{2} \text{ point}$$

➔ PLUS  $\frac{1}{2}$  point if you got at least 4 of the terms correct

Evaluate  $\sum_{m=3}^6 m(m-4)$ .

SCORE: \_\_\_\_ / 3 POINTS

**SHOW YOUR WORK. SIMPLIFY YOUR ANSWER.**

$$= 3(3-4) + 4(4-4) + 5(5-4) + 6(6-4)$$

$$= 3(-1) + 4(0) + 5(1) + 6(2)$$

$$= \boxed{-3} \quad \boxed{+0} \quad \boxed{+5} \quad \boxed{+12}$$

$\frac{1}{2}$  point  $\frac{1}{2}$  point  $\frac{1}{2}$  point  $\frac{1}{2}$  point

$$= \boxed{14}$$

1 point

Find the first 4 terms of the sequence defined recursively by  $a_1 = 2$ ,  $a_k = 3k - a_{k-1}$  (for  $k \geq 2$ ).

SCORE: \_\_\_\_ / 3 POINTS

$$a_1 = 2 \quad \text{➔ MINUS } \frac{1}{2} \text{ point if you forgot to write } a_1 = 2$$

$$a_2 = 3(2) - a_1 = 6 - 2 = \boxed{4} \quad 1 \text{ point}$$

$$a_3 = 3(3) - a_2 = 9 - 4 = \boxed{5} \quad 1 \text{ point}$$

$$a_4 = 3(4) - a_3 = 12 - 5 = \boxed{7} \quad 1 \text{ point}$$

Fill in the blanks: For the sum  $\sum_{k=2}^m a_k$ ,  $m$  is called the upper limit of summation,

SCORE: \_\_\_\_ / 2 POINTS

$k$  is called the index (OR dummy index) of summation, and

2 is called the lower limit of summation.

Find a general formula for the arithmetic sequence whose first term is 6, and whose sixth term is 14.

SCORE: \_\_\_\_ / 3 POINTS

**SHOW YOUR WORK.**

$$a_6 = a_1 + (6-1)d$$

$$14 = 6 + 5d$$

1 point

$$d = \frac{8}{5}$$

½ point

$$a_n = 6 + \frac{8}{5}(n-1)$$

1½ points

Use sigma notation to write the sum  $\frac{1}{4} + \frac{3}{8} + \frac{7}{16} + \frac{15}{32} + \frac{31}{64}$ .

SCORE: \_\_\_\_ / 3 POINTS

½ point

$$\sum_{n=1}^5$$

½ point

$$\sum_{n=1}^5 \frac{2^n - 1}{2^{n+1}}$$

½ point for numerator, ½ point for denominator ➔ PLUS ½ point if both correct

½ point

$$n=1$$

OR

½ point

$$\sum_{n=2}^6$$

½ point

$$\sum_{n=2}^6 \frac{2^{n-1} - 1}{2^n}$$

½ point for numerator, ½ point for denominator ➔ PLUS ½ point if both correct

½ point

$$n=2$$

Simplify the expression  $\frac{(4n-3)!}{(4n-1)!}$ .

SCORE: \_\_\_\_ / 3 POINTS

**SHOW YOUR WORK.**

$$= \frac{(4n-3) \cdots (3)(2)(1)}{(4n-1)(4n-2)(4n-3) \cdots (3)(2)(1)}$$

1½ points

OR

$$= \frac{(4n-3)!}{(4n-1)(4n-2)(4n-3)!}$$

1½ points

$$= \frac{1}{(4n-1)(4n-2)}$$

1½ points

$$= \frac{1}{(4n-1)(4n-2)}$$

1½ points