Fri Jan 13, 2012

SCORE: ___ / 20 POINTS

WHERE INDICATED, YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER TO GET FULL CREDIT.

Find the first 4 terms of the sequence defined recursively by $a_1 = 1, \quad a_k = k^2 - a_{k-1}$ (for $k \geq 2$). **SCORE: 3 / 3 POINTS**

$a_1 = 1, \ a_k = k^2 - a_{k-1}$ (for $k \geq 2$).

$a_2 = 2^2 - a_{2-1} = 2^2 - a_1 = 4 - 1 = 3$

$a_3 = 3^2 - a_{3-1} = 3^2 - a_2 = 9 - 3 = 6$

$a_4 = 4^2 - a_{4-1} = 4^2 - a_3 = 16 - 6 = 10$

The first 4 terms of the sequence defined recursively by $a_1 = 1, \ a_k = k^2 - a_{k-1}$ (for $k \geq 2$)

are $1, \underbrace{3}_{1}, \underbrace{6}_{1}, \underbrace{10}_{1}$.

Simplify the expression $\dfrac{(3n-3)!}{(3n-1)!}$. **SCORE: 3 / 3 POINTS**

SHOW YOUR WORK.

$$\frac{(3n-3)!}{(3n-1)!}$$

$$= \frac{\cancel{(3n-3)} \cdot \cancel{(3n-4)} \cdot \cancel{(3n-5)} \cdot \cdots \cdot \cancel{3} \cdot \cancel{2} \cdot 1}{(3n-1) \cdot (3n-2) \cdot \cancel{(3n-3)} \cdot \cancel{(3n-4)} \cdot \cancel{(3n-5)} \cdot \cdots \cdot \cancel{3} \cdot \cancel{2} \cdot 1} \quad \Big] \ 1\tfrac{1}{2}$$

$$= \underbrace{\frac{1}{(3n-1)(3n-2)}}_{1\frac{1}{2}}$$

Find a general formula for the arithmetic sequence whose first term is 6, and whose fourth term is 11. **SCORE: ___ / 3 POINTS**

SHOW YOUR WORK.

$a_n = a_1 + (n-1)d$

$a_1 = 6, \ a_4 = 11$

$a_4 = a_1 + (4-1)d$

$\underline{11 = 6 + 3d}$ 1

$3d = 5$

$\underline{d = \frac{5}{3}}$ $\frac{1}{2}$

$a_n = a_1 - d + nd$

$a_n = 6 - \frac{5}{3} + \frac{5}{3}n$

$a_n = \frac{13}{3} + \frac{5}{3}n$

∴ The general formula of this arithmetic sequence is $\underline{a_n = \frac{13}{3} + \frac{5}{3}n}$. *

SHOW YOUR WORK. SIMPLIFY YOUR ANSWER.

$$\sum_{m=2}^{5} m(m-3)$$

$$= 2(2-3) + 3(3-3) + 4(4-3) + 5(5-3)$$

$$= \underbrace{-2}_{\frac{1}{2}} + \underbrace{0}_{\frac{1}{2}} + \underbrace{4}_{\frac{1}{2}} + \underbrace{10}_{\frac{1}{2}}$$

$\frac{1}{2}$

$$= \underbrace{12}_{1}$$

Find the first 5 terms of the sequence defined by $a_n = \frac{1-(-1)^n}{n!}$. **SCORE:** 3 **/ 3 POINTS**

SIMPLIFY YOUR ANSWERS.

$$a_n = \frac{1-(-1)^n}{n!}$$

$$a_1 = \frac{1-(-1)^1}{1!} = 2$$

$$a_2 = \frac{1-(-1)^2}{2!} = 0$$

$$a_3 = \frac{1-(-1)^3}{3!} = \frac{2}{1\times2\times3} = \frac{1}{3}$$

$$a_4 = \frac{1-(-1)^4}{4!} = \frac{0}{4!} = 0$$

$$a_5 = \frac{1-(-1)^5}{5!} = \frac{2}{1\times2\times3\times4\times5} = \frac{1}{60}$$

$n=$	1	2	3	4	5
$a_n=$	2	0	$\frac{1}{3}$	0	$\frac{1}{60}$

The first 5 terms of the sequence defined by $a_n = \frac{1-(-1)^n}{n!}$ are $\underbrace{2}_{\frac{1}{2}}, \underbrace{0}_{\frac{1}{2}}, \underbrace{\frac{1}{3}}_{\frac{1}{2}}, \underbrace{0}_{\frac{1}{2}}, \underbrace{\frac{1}{60}}_{\frac{1}{2}}$.

$+\frac{1}{2}$ ALL 5 CORRECT

Fill in the blanks: For the sum $\sum_{m=2}^{k} a_m$, m is called the index of ~~the~~ summation, **SCORE:** 2 **/ 2 POINTS**

k is called the upper limit of ~~the~~ summation, and

2 is called the lower limit of ~~the~~ summation.

Use sigma notation to write the sum $\frac{1}{4} + \frac{3}{8} + \frac{7}{16} + \frac{15}{32} + \frac{31}{64}$. **SCORE:** 3 **/ 3 POINTS**

$n=$	1	2	3	4	5
$a_n=$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{15}{32}$	$\frac{31}{64}$

$$\frac{1}{4} + \frac{3}{8} + \frac{7}{16} + \frac{15}{32} + \frac{31}{64}$$

$$= \frac{2^1-1}{2^2} + \frac{2^2-1}{2^3} + \frac{2^3-1}{2^4} + \frac{2^4-1}{2^5} + \frac{2^5-1}{2^6}$$

$$\therefore a_n = \frac{2^n-1}{2^{n+1}}$$

The sigma notation of the sum $\frac{1}{4} + \frac{3}{8} + \frac{7}{16} + \frac{15}{32} + \frac{31}{64}$

is $\frac{1}{2}\left[\sum_{n=1}^{5}\left(\frac{2^n-1}{2^{n+1}}\right)\right]$ $\frac{1}{2}$ $\frac{1}{2}$ $+\frac{1}{2}$ BOTH NUMER + DENOM CORRECT

$\frac{1}{2}$ $\frac{1}{2}$