NAMES YOU ASKED TO BE CALLED IN CLASS:

_____

_____

SCORE: ___ / 30 POINTS

WHERE INDICATED, YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER TO GET FULL CREDIT.

IF A PROBLEM CAN BE SOLVED USING A SERIES FORMULA,
YOU MUST USE IT (NOT SIMPLY WRITE OUT EACH TERM AND ADD THEM TOGETHER)

Prove the following statement by mathematical induction.   **SCORE: ___ / 9 POINTS**

For all integers $n \geq 0$, $\quad \dfrac{1}{1 \times 6} + \dfrac{1}{6 \times 11} + \dfrac{1}{11 \times 16} + \cdots + \dfrac{1}{(5n+1) \times (5n+6)} = \dfrac{n+1}{5n+6}$

BASIS STEP: If $n = 0$, $\quad$ LEFT SIDE $= \dfrac{1}{1 \times 6} = \dfrac{1}{6} \quad$ RIGHT SIDE $= \dfrac{0+1}{5(0)+6} = \dfrac{1}{6}$

INDUCTIVE STEP: Assume that $\quad \dfrac{1}{1 \times 6} + \dfrac{1}{6 \times 11} + \dfrac{1}{11 \times 16} + \cdots + \dfrac{1}{(5k+1) \times (5k+6)} = \dfrac{k+1}{5k+6} \quad$ for some integer $k \geq 0$

Now prove $\quad \dfrac{1}{1 \times 6} + \dfrac{1}{6 \times 11} + \dfrac{1}{11 \times 16} + \cdots + \dfrac{1}{(5(k+1)+1) \times (5(k+1)+6)} = \dfrac{(k+1)+1}{5(k+1)+6}$

ie. $\quad \dfrac{1}{1 \times 6} + \dfrac{1}{6 \times 11} + \dfrac{1}{11 \times 16} + \cdots + \dfrac{1}{(5k+6) \times (5k+11)} = \dfrac{k+2}{5k+11}$

$\dfrac{1}{1 \times 6} + \dfrac{1}{6 \times 11} + \dfrac{1}{11 \times 16} + \cdots + \dfrac{1}{(5k+6) \times (5k+11)}$

$= \dfrac{1}{1 \times 6} + \dfrac{1}{6 \times 11} + \dfrac{1}{11 \times 16} + \cdots + \dfrac{1}{(5k+1) \times (5k+6)} + \dfrac{1}{(5k+6) \times (5k+11)}$

$= \dfrac{k+1}{5k+6} + \dfrac{1}{(5k+6)(5k+11)}$

$= \dfrac{(k+1)(5k+11)+1}{(5k+6)(5k+11)}$ ➡ **1 point for each statement/expression in a red box**

$= \dfrac{5k^2 + 16k + 12}{(5k+6)(5k+11)}$

$= \dfrac{(5k+6)(k+2)}{(5k+6)(5k+11)}$

$= \dfrac{k+2}{5k+11}$

By mathematical induction, $\dfrac{1}{1 \times 6} + \dfrac{1}{6 \times 11} + \dfrac{1}{11 \times 16} + \cdots + \dfrac{1}{(5n+1) \times (5n+6)} = \dfrac{n+1}{5n+6}$ for all integers $n \geq 0$.

Araceli bought a new car in 2004. Her car registration fee that year was $\$278$.   **SCORE: ___ / 5 POINTS**
Because of depreciation, the fee decreased by 12% each year (from the previous year).
How much did Araceli pay altogether in registration fees from 2004 until 2011 (including both those years)?

Total fees $= 278 \; + \; 278(1-0.12) \; + \; 278(1-0.12)^2 \; + \; \cdots \; + \; 278(1-0.12)^7$

$S_9 = \dfrac{278(1-0.88^8)}{1-0.88} = 1483.51$ **1 point**

4 points

After opening in January 2010, the number of patients seen each month at the Sao Christovao Clinic increased $\qquad$ **SCORE: ___ / 7 POINTS** by the same number every month. If 593 patients were seen during the 5th month, and 803 patients were seen during the 8th month, how many total patients were seen during the first 18 months ?

Let a_n = number of patients seen during n^{th} month

$a_8 - a_5 = (a_1 + 7d) - (a_1 + 4d) = 803 - 593$

$\boxed{3d = 210}$ **2 points**

$\boxed{d = 70}$ **1 point**

$a_1 + 4(70) = 593$

$\boxed{a_1 = 313}$ **1 point**

$S_{18} = \boxed{\dfrac{18}{2}[2(313) + 17(70)] \text{ or } \dfrac{18}{2}[313 + 1503]} = \boxed{16344}$ **1 point**

  2 points (either format is acceptable)

The third term of a geometric sequence is -300, and the sixth term is $19\dfrac{1}{5}$. **SCORE: ___ / 9 POINTS**

[a]   Find the tenth term. **DO NOT ROUND OFF YOUR ANSWER.**

$\dfrac{a_6}{a_3} = \dfrac{a_1 r^5}{a_1 r^2} = \dfrac{19\frac{1}{5}}{-300}$

$\boxed{r^3 = -0.064}$ **2 points**

$\boxed{r = -0.4}$ **1 point**

$a_1(-0.4)^2 = -300$

$\boxed{a_1 = -1875}$ **1½ points**

$a_{10} = -1875(-0.4)^9 = \boxed{0.49152}$ **1½ points**

[b]   Find the sum of the corresponding infinite series.

$S = \boxed{\dfrac{-1875}{1-(-0.4)}} = \boxed{-\dfrac{9375}{7} \text{ or } -1339.\overline{285714}}$ **1 point (either format is acceptable)**

  2 points