Math 43 (8:30am Class)
Quiz 3 Version J
Fri Feb 3, 2012

NAMES YOU ASKED TO BE CALLED IN CLASS:

____

____

SCORE: ___ / 30 POINTS

TO GET FULL CREDIT: **YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER**
YOU MUST USE THE STANDARD FORM FOR THE EQUATIONS AS SHOWN IN LECTURE AND THE TEXTBOOK

Consider the ellipse with equation $3x^2+5y^2+24x-20y+53=0$. **SCORE: ___ / 6 POINTS**

$$3x^2+24x+5y^2-20y=-53$$

$$3(x^2+8x)+5(y^2-4y)=-53$$

$$3(x^2+8x+16)+5(y^2-4y+4)=-53+3\cdot 16+5\cdot 4$$

½ ½ ½ ½ 1

½ $3(x+4)^2+5(y-2)^2=15$

½ $\dfrac{(x+4)^2}{5}+\dfrac{(y-2)^2}{3}=1$

[b] Find the co-ordinates of both vertices.

$(-4\pm\sqrt{5},\ 2)$

1

1 TOGETHER

Consider the ellipse with foci $(5,-10)$ and $(5,4)$ and a minor axis of length 16. **SCORE: ___ / 6 POINTS**

[a] Find the ends of the minor axis.

center $=\left(5,\dfrac{-10+4}{2}\right)=(5,-3)$ along vertical major axis 1

horizontal semi-minor axis $=\dfrac{16}{2}=8$ ½

ends of minor axis $=(5\pm 8,-3)=(13,-3)$ *and* $(-3,-3)$

½ ½

[b] Find the standard form of the equation of the ellipse.

focal length $=4-{}^{-}10=14$
½ focal length $=7$ ½
$a^2=8^2+7^2=113$
$a=\sqrt{113}$ 1 OR

$\dfrac{(x-5)^2}{64}+\dfrac{(y+3)^2}{113}=1$ 2

Find the standard form of the equation of the parabola with focus $(-9, 11)$ and directrix $x = 1$. **SCORE: ___ / 4 POINTS**

vertex $= \left(\dfrac{-9+1}{2}, 11\right) = (-4, 11)$

p = directed distance from $(-4, 11)$ to $(-9, 11) = -5$

vertical directrix

$$(y-11)^2 = 4(-5)(x - {}^{-}4)$$
$$(y-11)^2 = -20(x+4)$$

In this question, you will derive the formula for a hyperbola **using the distance-based definition** given in class. **SCORE: ___ / 9 POINTS**

Using the distance-based definition of a hyperbola,
find the standard form of the equation of the hyperbola containing all points whose distances to the foci $(0, \pm 6)$ differs by 2.

$$\sqrt{x^2+(y+6)^2} - \sqrt{x^2+(y-6)^2} = 2$$
$$\sqrt{x^2+(y+6)^2} = 2 + \sqrt{x^2+(y-6)^2}$$
$$x^2+(y+6)^2 = 4 + 4\sqrt{x^2+(y-6)^2} + x^2 + (y-6)^2$$
$$y^2+12y+36 = 4 + 4\sqrt{x^2+(y-6)^2} + y^2 - 12y + 36$$
$$24y - 4 = 4\sqrt{x^2+(y-6)^2}$$
$$6y - 1 = \sqrt{x^2+(y-6)^2}$$
$$36y^2 - 12y + 1 = x^2 + (y-6)^2$$
$$36y^2 - 12y + 1 = x^2 + y^2 - 12y + 36$$
$$35y^2 - x^2 = 35$$
$$y^2 - \frac{x^2}{35} = 1$$

OR

$$\sqrt{x^2+(y-6)^2} - \sqrt{x^2+(y+6)^2} = 2$$
$$\sqrt{x^2+(y-6)^2} = 2 + \sqrt{x^2+(y+6)^2}$$
$$x^2+(y-6)^2 = 4 + 4\sqrt{x^2+(y+6)^2} + x^2 + (y+6)^2$$
$$y^2-12y+36 = 4 + 4\sqrt{x^2+(y+6)^2} + y^2 + 12y + 36$$
$$-24y - 4 = 4\sqrt{x^2+(y+6)^2}$$
$$-6y - 1 = \sqrt{x^2+(y+6)^2}$$
$$36y^2 + 12y + 1 = x^2 + (y+6)^2$$
$$36y^2 + 12y + 1 = x^2 + y^2 + 12y + 36$$
$$35y^2 - x^2 = 35$$
$$y^2 - \frac{x^2}{35} = 1$$

Fill in the blanks. **SCORE: ___ / 5 POINTS**

[a] The **CONJUGATE** axis of a hyperbola passes through the center, but does not contain any points on the hyperbola.

[b] The difference of the distances between any point on a hyperbola and the foci equals the length of the **TRANSVERSE AXIS**.

MUST HAVE BOTH WORDS

[c] The eccentricity of an ellipse with $a = 20$ and $c = 5$ is $\dfrac{1}{4}$.

[d] The **VERTEX** of a parabola is the midpoint between the **FOCUS** and the **DIRECTRIX**.