SCORE: ___ / 30 POINTS

**TO GET FULL CREDIT: YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER
YOU MUST USE THE STANDARD FORM FOR THE EQUATIONS AS SHOWN
IN LECTURE AND THE TEXTBOOK**

Consider the ellipse with foci $(3, -12)$ and $(3, 6)$ and a minor axis of length 8.   SCORE: ___ / 6 POINTS

[a]   Find the ends of the minor axis.

$$\text{center} = \left(3, \frac{-12+6}{2}\right) = (3, -3) \text{ along vertical major axis}$$

$$\text{horizontal semi-minor axis} = \frac{8}{2} = 4$$

$$\text{ends of minor axis} = (3 \pm 4, -3) = (7, -3) \ and \ (-1, -3)$$

[b]   Find the standard form of the equation of the ellipse.

$$\text{focal length} = 6-^-12 = 18$$
$$\text{½ focal length} = 9$$
$$a^2 = 4^2 + 9^2 = 97$$
$$a = \sqrt{97} \quad or$$

$$\frac{(x-3)^2}{16} + \frac{(y+3)^2}{97} = 1$$

Consider the ellipse with equation $5x^2+3y^2+20x-24y+53 = 0$.   SCORE: ___ / 6 POINTS

[a]   Find the standard form of the equation of the ellipse.

$$5x^2+20x + 3y^2-24y = -53$$
$$5(x^2+4x) + 3(y^2-8y) = -53$$
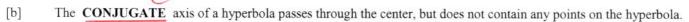$$5(x^2+4x+4) + 3(y^2-8y+16) = -53 + 5\cdot4 + 3\cdot16$$
$$5(x+2)^2 + 3(y-4)^2 = 15$$
$$\frac{(x+2)^2}{3} + \frac{(y-4)^2}{5} = 1$$

[b]   Find the co-ordinates of both vertices.

$$(-2, 4\pm\sqrt{5})$$

Fill in the blanks.   **SCORE: ___ / 5 POINTS**

[a]   The eccentricity of an ellipse with $a = 20$ and $c = 4$ is $\dfrac{1}{5}$. ①

        ①

[b]   The **CONJUGATE** axis of a hyperbola passes through the center, but does not contain any points on the hyperbola.

[c]   The **VERTEX** of a parabola is the midpoint between the **FOCUS** and the **DIRECTRIX** ①
        ①

                                                         MUST HAVE BOTH WORDS

[d]   The difference of the distances between any point on a hyperbola and the foci equals the length of the **TRANSVERSE AXIS**. ①

In this question, you will derive the formula for a hyperbola **using the distance-based definition** given in class.   **SCORE: ___ / 9 POINTS**

     Using the distance-based definition of a hyperbola,
     find the standard form of the equation of the hyperbola containing all points whose distances to the foci $(0, \pm 5)$ differs by 2.

① $\sqrt{x^2 + (y+5)^2} - \sqrt{x^2 + (y-5)^2} = 2$

① $\sqrt{x^2 + (y+5)^2} = 2 + \sqrt{x^2 + (y-5)^2}$

① $x^2 + (y+5)^2 = 4 + 4\sqrt{x^2 + (y-5)^2} + x^2 + (y-5)^2$

$y^2 + 10y + 25 = 4 + 4\sqrt{x^2 + (y-5)^2} + y^2 - 10y + 25$

① $20y - 4 = 4\sqrt{x^2 + (y-5)^2}$

① $5y - 1 = \sqrt{x^2 + (y-5)^2}$

$25y^2 - 10y + 1 = x^2 + (y-5)^2$

① $25y^2 - 10y + 1 = x^2 + y^2 - 10y + 25$

① $24y^2 - x^2 = 24$

② $y^2 - \dfrac{x^2}{24} = 1$

OR ① $\sqrt{x^2 + (y-5)^2} - \sqrt{x^2 + (y+5)^2} = 2$

① $\sqrt{x^2 + (y-5)^2} = 2 + \sqrt{x^2 + (y+5)^2}$

① $x^2 + (y-5)^2 = 4 + 4\sqrt{x^2 + (y+5)^2} + x^2 + (y+5)^2$

$y^2 - 10y + 25 = 4 + 4\sqrt{x^2 + (y+5)^2} + y^2 + 10y + 25$

① $-20y - 4 = 4\sqrt{x^2 + (y+5)^2}$

① $-5y - 1 = \sqrt{x^2 + (y+5)^2}$

$25y^2 + 10y + 1 = x^2 + (y+5)^2$

① $25y^2 + 10y + 1 = x^2 + y^2 + 10y + 25$

① $24y^2 - x^2 = 24$

② $y^2 - \dfrac{x^2}{24} = 1$

Find the standard form of the equation of the parabola with focus $(-7, 9)$ and directrix $x = 1$.   **SCORE: ___ / 4 POINTS**

$\text{vertex} = \left(\dfrac{-7+1}{2}, 9 \right) = (-3, 9)$ (1½)

p = directed distance from $(-3, 9)$ to $(-7, 9) = -4$
vertical directrix

$(y - 9)^2 = 4(-4)(x - {}^-3)$
$(y - 9)^2 = -16(x + 3)$
       ①   ①   (½)