Math 43 (8:30am Class)
Quiz 3 Version P
Fri Feb 3, 2012

NAMES YOU ASKED TO BE CALLED IN CLASS:

_____

_____

SCORE: ___ / 30 POINTS

TO GET FULL CREDIT: YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER
YOU MUST USE THE STANDARD FORM FOR THE EQUATIONS AS SHOWN IN LECTURE AND THE TEXTBOOK

Fill in the blanks. **SCORE: ___ / 5 POINTS**

[a] The **VERTEX** of a parabola is the midpoint between the **FOCUS** and the **DIRECTRIX** . (1) (1)

[b] The eccentricity of an ellipse with $a = 6$ and $c = 3$ is $\frac{1}{2}$. (1)

MUST HAVE BOTH WORDS

[c] The difference of the distances between any point on a hyperbola and the foci equals the length of the **TRANSVERSE AXIS** (1)

[d] The **CONJUGATE** axis of a hyperbola passes through the center, but does not contain any points on the hyperbola. (1)

Find the standard form of the equation of the parabola with focus $(-3, 5)$ and directrix $x = 1$. **SCORE: ___ / 4 POINTS**

vertex $= \left(\frac{-3+1}{2}, 5\right) = (-1, 5)$ (1 1/2)

p = directed distance from $(-1, 5)$ to $(-3, 5) = -2$
vertical directrix

$(y-5)^2 = 4(-2)(x-^-1)$
$(y-5)^2 = -8(x+1)$
(1) (1) (1/2)

Consider the ellipse with foci $(2, -11)$ and $(2, 5)$ and a minor axis of length 12. **SCORE: ___ / 6 POINTS**

[a] Find the ends of the minor axis.

center $= \left(2, \frac{-11+5}{2}\right) = (2, -3)$ along vertical major axis (1)

horizontal semi-minor axis $= \frac{12}{2} = 6$ (1/2)

ends of minor axis $= (2 \pm 6, -3) = (8, -3)$ *and* $(-4, -3)$
(1/2) (1/2)

[b] Find the standard form of the equation of the ellipse.

focal length $= 5-^-11 = 16$
½ focal length $= 8$ (1/2)
$a^2 = 6^2 + 8^2 = 100$
$a = 10$ (1) OR

$\frac{(x-2)^2}{36} + \frac{(y+3)^2}{100} = 1$ (2)

Consider the ellipse with equation $5x^2+3y^2-40x+12y+77=0$. **SCORE: ___ / 6 POINTS**

[a] Find the standard form of the equation of the ellipse.

$$5x^2-40x+3y^2+12y=-77$$
$$5(x^2-8x)+3(y^2+4y)=-77$$
$$5(x^2-8x+16)+3(y^2+4y+4)=-77+5\cdot 16+3\cdot 4$$
$$5(x-4)^2+3(y+2)^2=15$$
$$\frac{(x-4)^2}{3}+\frac{(y+2)^2}{5}=1$$

[b] Find the co-ordinates of both vertices.

$$(4,-2\pm\sqrt{5})$$

In this question, you will derive the formula for a hyperbola **using the distance-based definition** given in class. **SCORE: ___ / 9 POINTS**

Using the distance-based definition of a hyperbola,
find the standard form of the equation of the hyperbola containing all points whose distances to the foci $(0,\pm 3)$ differs by 2.

$$\sqrt{x^2+(y+3)^2}-\sqrt{x^2+(y-3)^2}=2$$
$$\sqrt{x^2+(y+3)^2}=2+\sqrt{x^2+(y-3)^2}$$
$$x^2+(y+3)^2=4+4\sqrt{x^2+(y-3)^2}+x^2+(y-3)^2$$
$$y^2+6y+9=4+4\sqrt{x^2+(y-3)^2}+y^2-6y+9$$
$$12y-4=4\sqrt{x^2+(y-3)^2}$$
$$3y-1=\sqrt{x^2+(y-3)^2}$$
$$9y^2-6y+1=x^2+(y-3)^2$$
$$9y^2-6y+1=x^2+y^2-6y+9$$
$$8y^2-x^2=8$$
$$y^2-\frac{x^2}{8}=1$$

OR

$$\sqrt{x^2+(y-3)^2}-\sqrt{x^2+(y+3)^2}=2$$
$$\sqrt{x^2+(y-3)^2}=2+\sqrt{x^2+(y+3)^2}$$
$$x^2+(y-3)^2=4+4\sqrt{x^2+(y+3)^2}+x^2+(y+3)^2$$
$$y^2-6y+9=4+4\sqrt{x^2+(y-3)^2}+y^2+6y+9$$
$$-12y-4=4\sqrt{x^2+(y+3)^2}$$
$$-3y-1=\sqrt{x^2+(y+3)^2}$$
$$9y^2+6y+1=x^2+(y+3)^2$$
$$9y^2+6y+1=x^2+y^2+6y+9$$
$$8y^2-x^2=8$$
$$y^2-\frac{x^2}{8}=1$$