Math 43 (8:30am Class)
Quiz 3 Version R
Fri Feb 3, 2012

NAMES YOU ASKED TO BE CALLED IN CLASS:

____

____

SCORE: ___ / 30 POINTS

TO GET FULL CREDIT: **YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER**
YOU MUST USE THE STANDARD FORM FOR THE EQUATIONS AS SHOWN IN LECTURE AND THE TEXTBOOK

Find the standard form of the equation of the parabola with focus $(-5, 7)$ and directrix $x = 1$. **SCORE: ___ / 4 POINTS**

vertex $= \left(\dfrac{-5+1}{2}, 7\right) = \underline{(-2, 7)}$ $1\frac{1}{2}$

p = directed distance from $(-2, 7)$ to $(-5, 7) = -3$
vertical directrix

$$(y-7)^2 = 4(-3)(x-{}^{-}2)$$
$$\underset{1}{\underline{(y-7)^2}} = \underset{1}{\underline{-12}}\underset{\frac{1}{2}}{\underline{(x+2)}}$$

Consider the ellipse with equation $3x^2+5y^2-12x+40y+77=0$. **SCORE: ___ / 6 POINTS**

[a] Find the standard form of the equation of the ellipse.

$$3x^2-12x+5y^2+40y=-77$$
$$3(x^2-4x)+5(y^2+8y)=-77 \quad \tfrac{1}{2}$$
$$\tfrac{1}{2}\ \underline{3(x^2-4x+4)}+\underline{5(y^2+8y+16)}=\underline{-77+3\cdot 4+5\cdot 16} \quad 1$$
$$\underline{3(x-2)^2+5(y+4)^2=15} \quad \tfrac{1}{2}$$
$$\underline{\frac{(x-2)^2}{5}+\frac{(y+4)^2}{3}=1} \quad \tfrac{1}{2}$$

[b] Find the co-ordinates of both vertices.

$$(2 \underset{1}{\underline{\pm\sqrt{5}}}, -4)$$

1 TOGETHER

Fill in the blanks. **SCORE: ___ / 5 POINTS**

MUST HAVE BOTH WORDS

[a] The difference of the distances between any point on a hyperbola and the foci equals the length of the **TRANSVERSE AXIS** . 1

[b] The **VERTEX** 1 of a parabola is the midpoint between the **FOCUS** and the **DIRECTRIX** . 1

[c] The **CONJUGATE** 1 axis of a hyperbola passes through the center, but does not contain any points on the hyperbola.

[d] The eccentricity of an ellipse with $a = 6$ and $c = 2$ is $\dfrac{1}{3}$. 1

In this question, you will derive the formula for a hyperbola **using the distance-based definition** given in class. **SCORE: ___ / 9 POINTS**

Using the distance-based definition of a hyperbola,
find the standard form of the equation of the hyperbola containing all points whose distances to the foci $(0, \pm 4)$ differs by 2.

1 $\sqrt{x^2+(y+4)^2}-\sqrt{x^2+(y-4)^2}=2$

1 $\sqrt{x^2+(y+4)^2}=2+\sqrt{x^2+(y-4)^2}$

1 $x^2+(y+4)^2=4+4\sqrt{x^2+(y-4)^2}+x^2+(y-4)^2$

$y^2+8y+16=4+4\sqrt{x^2+(y-4)^2}+y^2-8y+16$

1 $16y-4=4\sqrt{x^2+(y-4)^2}$

1 $4y-1=\sqrt{x^2+(y-4)^2}$

$16y^2-8y+1=x^2+(y-4)^2$

1 $16y^2-8y+1=x^2+y^2-8y+16$

1 $15y^2-x^2=15$

2 $y^2-\dfrac{x^2}{15}=1$

OR

1 $\sqrt{x^2+(y-4)^2}-\sqrt{x^2+(y+4)^2}=2$

1 $\sqrt{x^2+(y-4)^2}=2+\sqrt{x^2+(y+4)^2}$

1 $x^2+(y-4)^2=4+4\sqrt{x^2+(y+4)^2}+x^2+(y+4)^2$

$y^2-8y+16=4+4\sqrt{x^2+(y+4)^2}+y^2+8y+16$

1 $-16y-4=4\sqrt{x^2+(y+4)^2}$

1 $-4y-1=\sqrt{x^2+(y+4)^2}$

$16y^2+8y+1=x^2+(y+4)^2$

1 $16y^2+8y+1=x^2+y^2+8y+16$

1 $15y^2-x^2=15$

2 $y^2-\dfrac{x^2}{15}=1$

Consider the ellipse with foci $(4, -10)$ and $(4, 2)$ and a minor axis of length 10. **SCORE: ___ / 6 POINTS**

[a] Find the ends of the minor axis.

center $=\left(4, \dfrac{-10+2}{2}\right)=(4, -4)$ along vertical major axis 1

horizontal semi-minor axis $=\dfrac{10}{2}=5$ ½

ends of minor axis $=(4\pm 5, -4)=(9, -4)$ *and* $(-1, -4)$ ½ ½

[b] Find the standard form of the equation of the ellipse.

focal length $=2-{}^{-}10=12$
½ focal length $=6$ ½
$a^2=5^2+6^2=61$
$a=\sqrt{61}$ 1 OR

$$\frac{(x-4)^2}{25}+\frac{(y+4)^2}{61}=1$$ 2