

SCORE: ____ / 30 POINTS

TO GET FULL CREDIT:

YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER
YOU MUST USE THE STANDARD FORM FOR THE EQUATIONS AS SHOWN
IN LECTURE AND THE TEXTBOOK

Fill in the blanks.

SCORE: ____ / 5 POINTS

[a] The VERTEX of a parabola is the midpoint between the FOCUS and the DIRECTRIX.

[b] The eccentricity of an ellipse with $a = 6$ and $c = 3$ is $\frac{1}{2}$.

MUST HAVE BOTH
↓
WORDS

[c] The difference of the distances between any point on a hyperbola and the foci equals the length of the TRANSVERSE AXIS.

[d] The CONJUGATE axis of a hyperbola passes through the center, but does not contain any points on the hyperbola.

Find the standard form of the equation of the parabola with focus $(-3, 5)$ and directrix $x = 1$.

SCORE: ____ / 4 POINTS

$$\text{vertex} = \left(\frac{-3+1}{2}, 5 \right) = (-1, 5)$$

$p =$ directed distance from $(-1, 5)$ to $(-3, 5) = -2$

vertical directrix

$$(y-5)^2 = 4(-2)(x-1)$$

$$(y-5)^2 = -8(x+1)$$

Consider the ellipse with foci $(2, -11)$ and $(2, 5)$ and a minor axis of length 12.

SCORE: ____ / 6 POINTS

[a] Find the ends of the minor axis.

$$\text{center} = \left(2, \frac{-11+5}{2} \right) = (2, -3)$$

$$\text{horizontal semi-minor axis} = \frac{12}{2} = 6$$

$$\text{ends of minor axis} = (2 \pm 6, -3) = (8, -3) \text{ and } (-4, -3)$$

[b] Find the standard form of the equation of the ellipse.

$$\text{focal length} = 5 - (-11) = 16$$

$$\frac{1}{2} \text{ focal length} = 8$$

$$a^2 = 6^2 + 8^2 = 100$$

$$a = 10$$

$$\frac{(x-2)^2}{36} + \frac{(y+3)^2}{100} = 1$$

Consider the ellipse with equation $5x^2 + 3y^2 - 40x + 12y + 77 = 0$.

SCORE: ____ / 6 POINTS

- [a] Find the standard form of the equation of the ellipse.

$$\begin{aligned}
 5x^2 - 40x + 3y^2 + 12y &= -77 \\
 5(x^2 - 8x) + 3(y^2 + 4y) &= -77 \\
 \left(\frac{1}{2}\right) 5(x^2 - 8x + 16) + 3(y^2 + 4y + 4) &= -77 + 5 \cdot 16 + 3 \cdot 4 \quad (1) \\
 \left(\frac{1}{2}\right) 5(x-4)^2 + 3(y+2)^2 &= 15 \quad (1) \\
 \frac{(x-4)^2}{3} + \frac{(y+2)^2}{5} &= 1 \\
 \left(\frac{1}{2}\right) \frac{(x-4)^2}{3} + \frac{(y+2)^2}{5} &= 1
 \end{aligned}$$

- [b] Find the co-ordinates of both vertices.

$$(4, -2 \pm \sqrt{5})$$

In this question, you will derive the formula for a hyperbola **using the distance-based definition** given in class.

SCORE: ____ / 9 POINTS

Using the distance-based definition of a hyperbola,

find the standard form of the equation of the hyperbola containing all points whose distances to the foci $(0, \pm 3)$ differs by 2.

$$\begin{aligned}
 \sqrt{x^2 + (y+3)^2} - \sqrt{x^2 + (y-3)^2} &= 2 \\
 \sqrt{x^2 + (y+3)^2} &= 2 + \sqrt{x^2 + (y-3)^2} \\
 x^2 + (y+3)^2 &= 4 + 4\sqrt{x^2 + (y-3)^2} + x^2 + (y-3)^2 \\
 y^2 + 6y + 9 &= 4 + 4\sqrt{x^2 + (y-3)^2} + y^2 - 6y + 9 \\
 12y - 4 &= 4\sqrt{x^2 + (y-3)^2} \\
 3y - 1 &= \sqrt{x^2 + (y-3)^2} \\
 9y^2 - 6y + 1 &= x^2 + (y-3)^2 \\
 9y^2 - 6y + 1 &= x^2 + y^2 - 6y + 9 \\
 8y^2 - x^2 &= 8 \\
 y^2 - \frac{x^2}{8} &= 1
 \end{aligned}$$

OR

$$\begin{aligned}
 \sqrt{x^2 + (y-3)^2} - \sqrt{x^2 + (y+3)^2} &= 2 \\
 \sqrt{x^2 + (y-3)^2} &= 2 + \sqrt{x^2 + (y+3)^2} \\
 x^2 + (y-3)^2 &= 4 + 4\sqrt{x^2 + (y+3)^2} + x^2 + (y+3)^2 \\
 y^2 - 6y + 9 &= 4 + 4\sqrt{x^2 + (y+3)^2} + y^2 + 6y + 9 \\
 -12y - 4 &= 4\sqrt{x^2 + (y+3)^2} \\
 -3y - 1 &= \sqrt{x^2 + (y+3)^2} \\
 9y^2 + 6y + 1 &= x^2 + (y+3)^2 \\
 9y^2 + 6y + 1 &= x^2 + y^2 + 6y + 9 \\
 8y^2 - x^2 &= 8 \\
 y^2 - \frac{x^2}{8} &= 1
 \end{aligned}$$