SCORE: ___/30 POINTS

TO GET FULL CREDIT:

YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER YOU MUST USE THE STANDARD FORM FOR THE EQUATIONS AS SHOWN IN LECTURE AND THE TEXTBOOK

Consider the ellipse with equation $3x^2+5y^2+24x-20y+53=0$.

SCORE: ___ / 6 POINTS

$$3x^{2}+24x+5y^{2}-20y = -53$$

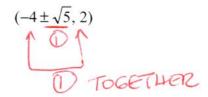
$$3(x^{2}+8x)+5(y^{2}-4y) = -53$$

$$3(x^{2}+8x+16)+5(y^{2}-4y+4) = -53+3\cdot16+5\cdot4$$

$$3(x+4)^{2}+5(y-2)^{2}=15$$

$$\frac{(x+4)^{2}}{5}+\frac{(y-2)^{2}}{3}=1$$

[b] Find the co-ordinates of both vertices.



Consider the ellipse with foci (5, -10) and (5, 4) and a minor axis of length 16.

SCORE: ___ / 6 POINTS

[a] Find the ends of the minor axis.

center
$$=$$
 $\left(5, \frac{-10+4}{2}\right) = \underbrace{(5, -3)}_{\text{along vertical major axis}}$
horizontal semi-minor axis $=\frac{16}{2} = \underbrace{8}_{\text{em}}$

ends of minor axis =
$$(5 \pm 8, -3) = \underbrace{(13, -3)}_{\text{ }}$$
 and $\underbrace{(-3, -3)}_{\text{ }}$

[b] Find the standard form of the equation of the ellipse.

focal length =
$$4 - 10 = 14$$

½ focal length = 7
 $a^2 = 8^2 + 7^2 = 113$
 $a = \sqrt{113}$

$$\frac{(x-5)^2}{64} + \frac{(y+3)^2}{113} = 1$$

vertex =
$$\left(\frac{-9+1}{2}, 11\right) = \underline{(-4, 11)}$$

p = directed distance from (-4, 11) to (-9, 11) = -5vertical directrix

$$(y-11)^{2} = 4(-5)(x-4)$$

$$(y-11)^{2} = -20(x+4)$$

In this question, you will derive the formula for a hyperbola using the distance-based definition given in class. SCORE: / 9 POINTS

Using the distance-based definition of a hyperbola,

find the standard form of the equation of the hyperbola containing all points whose distances to the foci $(0, \pm 6)$ differs by 2.

$$\int \sqrt{x^2 + (y+6)^2} - \sqrt{x^2 + (y-6)^2} = 2$$

$$\int \sqrt{x^2 + (y+6)^2} = 2 + \sqrt{x^2 + (y-6)^2}$$

$$\int x^2 + (y+6)^2 = 4 + 4\sqrt{x^2 + (y-6)^2} + x^2 + (y-6)^2$$

$$y^2 + 12y + 36 = 4 + 4\sqrt{x^2 + (y-6)^2} + y^2 - 12y + 36$$

$$\int 24y - 4 = 4\sqrt{x^2 + (y-6)^2}$$

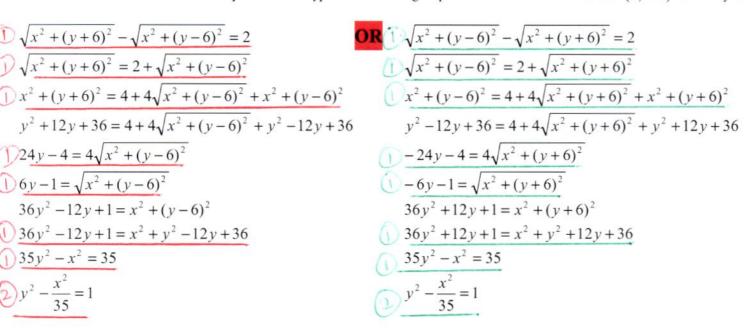
$$\int 6y - 1 = \sqrt{x^2 + (y-6)^2}$$

$$36y^2 - 12y + 1 = x^2 + (y-6)^2$$

$$\int 36y^2 - 12y + 1 = x^2 + y^2 - 12y + 36$$

$$\int 35y^2 - x^2 = 35$$

$$\int y^2 - \frac{x^2}{35} = 1$$



Fill in the blanks.

SCORE: ___ / 5 POINTS

- The CONJUGATE axis of a hyperbola passes through the center, but does not contain any points on the hyperbola. [a]
- The difference of the distances between any point on a hyperbola and the foci equals the length of the TRANSVERSE AXIS [6]
- The eccentricity of an ellipse with a = 20 and c = 5 is $\frac{1}{4}$ [c]

BOTH WORDS

The VERTEX of a parabola is the midpoint between the FOCUS and the DIRECTRIX [d]