Fri Feb 3, 2012

SCORE: ___ / 30 POINTS

TO GET FULL CREDIT: **YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER**
YOU MUST USE THE STANDARD FORM FOR THE EQUATIONS AS SHOWN IN LECTURE AND THE TEXTBOOK

Consider the ellipse with equation $3x^2+5y^2+24x-20y+53=0$. **SCORE: ___ / 6 POINTS**

$$3x^2+24x+5y^2-20y=-53$$
$$3(x^2+8x)+5(y^2-4y)=-53$$
$$3(x^2+8x+16)+5(y^2-4y+4)=-53+3\cdot 16+5\cdot 4$$
$$3(x+4)^2+5(y-2)^2=15$$
$$\frac{(x+4)^2}{5}+\frac{(y-2)^2}{3}=1$$

[b] Find the co-ordinates of both vertices.

$$(-4\pm\sqrt{5},\ 2)$$

Consider the ellipse with foci $(5,-10)$ and $(5,4)$ and a minor axis of length 16. **SCORE: ___ / 6 POINTS**

[a] Find the ends of the minor axis.

$$\text{center} = \left(5, \frac{-10+4}{2}\right) = (5,-3) \text{ along vertical major axis}$$

$$\text{horizontal semi-minor axis} = \frac{16}{2} = 8$$

$$\text{ends of minor axis} = (5\pm 8, -3) = (13,-3) \text{ and } (-3,-3)$$

[b] Find the standard form of the equation of the ellipse.

$$\text{focal length} = 4-{}^{-}10 = 14$$
$$\text{½ focal length} = 7$$
$$a^2 = 8^2+7^2 = 113$$
$$a = \sqrt{113}$$

$$\frac{(x-5)^2}{64}+\frac{(y+3)^2}{113}=1$$

Find the standard form of the equation of the parabola with focus $(-9, 11)$ and directrix $x = 1$. **SCORE: ___ / 4 POINTS**

vertex $= \left(\frac{-9+1}{2}, 11\right) = (-4, 11)$ (1½)

p = directed distance from $(-4, 11)$ to $(-9, 11) = -5$

vertical directrix

$(y-11)^2 = 4(-5)(x-^-4)$

$(y-11)^2 = -20(x+4)$

(1) (1) (½)

In this question, you will derive the formula for a hyperbola **using the distance-based definition** given in class. **SCORE: ___ / 9 POINTS**

Using the distance-based definition of a hyperbola,
find the standard form of the equation of the hyperbola containing all points whose distances to the foci $(0, \pm 6)$ differs by 2.

(1) $\sqrt{x^2+(y+6)^2} - \sqrt{x^2+(y-6)^2} = 2$

(1) $\sqrt{x^2+(y+6)^2} = 2 + \sqrt{x^2+(y-6)^2}$

(1) $x^2+(y+6)^2 = 4 + 4\sqrt{x^2+(y-6)^2} + x^2+(y-6)^2$

$y^2+12y+36 = 4 + 4\sqrt{x^2+(y-6)^2} + y^2-12y+36$

(1) $24y - 4 = 4\sqrt{x^2+(y-6)^2}$

(1) $6y - 1 = \sqrt{x^2+(y-6)^2}$

$36y^2-12y+1 = x^2+(y-6)^2$

(1) $36y^2-12y+1 = x^2+y^2-12y+36$

(1) $35y^2 - x^2 = 35$

(2) $y^2 - \frac{x^2}{35} = 1$

OR

(1) $\sqrt{x^2+(y-6)^2} - \sqrt{x^2+(y+6)^2} = 2$

(1) $\sqrt{x^2+(y-6)^2} = 2 + \sqrt{x^2+(y+6)^2}$

(1) $x^2+(y-6)^2 = 4 + 4\sqrt{x^2+(y+6)^2} + x^2+(y+6)^2$

$y^2-12y+36 = 4 + 4\sqrt{x^2+(y+6)^2} + y^2+12y+36$

(1) $-24y - 4 = 4\sqrt{x^2+(y+6)^2}$

(1) $-6y - 1 = \sqrt{x^2+(y+6)^2}$

$36y^2+12y+1 = x^2+(y+6)^2$

(1) $36y^2+12y+1 = x^2+y^2+12y+36$

(1) $35y^2 - x^2 = 35$

(2) $y^2 - \frac{x^2}{35} = 1$

Fill in the blanks. (1) **SCORE: ___ / 5 POINTS**

[a] The **CONJUGATE** axis of a hyperbola passes through the center, but does not contain any points on the hyperbola.

[b] The difference of the distances between any point on a hyperbola and the foci equals the length of the **TRANSVERSE AXIS**. (1)

MUST HAVE BOTH WORDS

[c] The eccentricity of an ellipse with $a = 20$ and $c = 5$ is $\frac{1}{4}$. (1)

[d] The **VERTEX** of a parabola is the midpoint between the **FOCUS** and the **DIRECTRIX**. (1) (1)