Fri Feb 10, 2012

SCORE: ___ / 30 POINTS

TO GET FULL CREDIT: YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER

Test $r = \sin\theta + \cos\theta$ for symmetry with respect to the polar axis. State clearly the conclusion of the test. **SCORE: ___ / 4 POINTS**

1 $r = \sin(-\theta) + \cos(-\theta)$

$\frac{1}{2}$ $r = -\sin\theta + \cos\theta$

$-r = \sin(\pi - \theta) + \cos(\pi - \theta)$ 1

$-r = \sin\pi\cos\theta - \cos\pi\sin\theta$

$+ \cos\pi\cos\theta + \sin\pi\sin\theta$

$-r = \sin\theta - \cos\theta$

$\frac{1}{2}$ $r = -\sin\theta + \cos\theta$

1 NO CONCLUSION

Find all values of θ (for $0 \le \theta < 2\pi$) where the graph of $r = 3 + 6\sin\theta$ passes through the pole. **SCORE: ___ / 3 POINTS**

1 $0 = 3 + 6\sin\theta$

$\frac{1}{2}$ $\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

1 $\frac{1}{2}$

Convert the rectangular equation $xy = 6$ to polar form. Simplify your final answer using identities. **SCORE: ___ / 3 POINTS**

1 $r\cos\theta \, r\sin\theta = 6$

$1\frac{1}{2}$ $r^2 = \frac{6}{\sin\theta\cos\theta}$

$r^2 = \frac{12}{2\sin\theta\cos\theta} = \frac{12}{\sin 2\theta} = 12\csc 2\theta$ $\frac{1}{2}$

Convert the polar equation $r^2 = \cos 2\theta$ to rectangular form. **SCORE: ___ / 4 POINTS**
Your final answer must NOT have radicals, but may use factored expressions.

2 $r^2 = \cos^2\theta - \sin^2\theta$

1 $r^4 = r^2\cos^2\theta - r^2\sin^2\theta$

1 $(x^2+y^2)^2 = x^2 - y^2$

OR

$r^2 = 1 - 2\sin^2\theta$ 2

$r^4 = r^2 - 2r^2\sin^2\theta$ 1

$(x^2+y^2)^2 = x^2+y^2-2y^2$ $\frac{1}{2}$

$(x^2+y^2)^2 = x^2-y^2$ $\frac{1}{2}$

Convert the rectangular co-ordinates $(-\sqrt{3}, -3)$ to polar co-ordinates using $r > 0$ and $0 \le \theta < 2\pi$. **SCORE: ___ / 2 POINTS**

NO POINTS, BUT MINUS $\frac{1}{2}$ IF MISSING $\rightarrow$ $\left(2\sqrt{3}, \frac{4\pi}{3}\right)$

$r = \sqrt{(-\sqrt{3})^2 + (-3)^2}$

$r = \sqrt{12} = 2\sqrt{3}$ $\frac{1}{2}$

$\theta = \pi + \tan^{-1}\frac{-3}{-\sqrt{3}}$

$\theta = \pi + \tan^{-1}\sqrt{3}$ $\frac{1}{2}$ $= \pi + \frac{\pi}{3}$ $\frac{1}{2}$ $= \frac{4\pi}{3}$ $\frac{1}{2}$

Fill in the blanks. **SCORE: ___ / 5 POINTS**

[a] The asymptotes of a hyperbola intersect at the CENTER (1) of the hyperbola.

[b] If replacing (r, θ) in a polar equation with $(-r, \pi - \theta)$ yields an equivalent equation, then the graph of the equation is symmetric with respect to THE POLAR AXIS (1).

[c] If the point with rectangular co-ordinates $(0, -7)$ has polar co-ordinates $(7, \theta)$ and $0 \le \theta < 2\pi$, then $\theta =$ $\frac{3\pi}{2}$ (1).

[d] In the polar co-ordinate system, the locus of points with co-ordinates $(0, \theta)$ is called THE POLE (1).

[e] The conic with equation $109x^2 + 109x - 97y^2 + 253y - 671 = 0$ is a/an HYPERBOLA (1).

A hyperbola has asymptotes $2x - y - 4 = 0$ and $2x + y - 4 = 0$. If one of the foci is at $(-3, 0)$, find the equation of the hyperbola. **SCORE: ___ / 7 POINTS**

NOTE: $a = h$, $b = v$

$y = 2x - 4$
$y = -2x + 4$ $\quad m = \pm 2$

$2x - 4 = -2x + 4$

$4x = 8$

$x = 2$

$y = 0$

CENTER $(2, 0)$ (1)

SEMI-FOCAL LENGTH $= 2 - -3 = 5$

HORIZONTAL TRANSVERSE AXIS

$\frac{v}{h} = 2$ (1)

$v = 2h$

$v^2 + h^2 = 5^2$ (1)

$4h^2 + h^2 = 25$

$5h^2 = 25$ (1)

$h^2 = 5$

$h = \sqrt{5}$ $\frac{1}{2}$

$v = 2\sqrt{5}$ $\frac{1}{2}$

$$\frac{(x-2)^2}{5} - \frac{y^2}{20} = 1$$ (2)

A point has polar co-ordinates $\left(7, \frac{4\pi}{5}\right)$. **SCORE: ___ / 2 POINTS**

[a] Find another polar representation for this point using $r > 0$ and $-2\pi < \theta < 2\pi$.

$\left(7, \frac{4\pi}{5} - 2\pi\right) = \left(7, -\frac{6\pi}{5}\right)$ (1)

[b] Find another polar representation for this point using $r < 0$ and $-2\pi < \theta < 2\pi$.

$\left(-7, \frac{4\pi}{5} + \pi\right) = \left(-7, \frac{9\pi}{5}\right)$ (1) OR $\left(-7, \frac{4\pi}{5} - \pi\right) = \left(-7, -\frac{\pi}{5}\right)$ (1)