Fri Feb 10, 2012

SCORE: ___ / 30 POINTS

TO GET FULL CREDIT: YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER

Test $r = \sin\theta + \cos\theta$ for symmetry with respect to $\theta = \frac{\pi}{2}$. State clearly the conclusion of the test. **SCORE: ___ / 4 POINTS**

$-r = \sin(-\theta) + \cos(-\theta)$ ①

$-r = -\sin\theta + \cos\theta$

$r = \sin\theta - \cos\theta$ ($\frac{1}{2}$)

① $r = \sin(\pi - \theta) + \cos(\pi - \theta)$

$r = \sin\pi\cos\theta - \cos\pi\sin\theta + \cos\pi\cos\theta + \sin\pi\sin\theta$

($\frac{1}{2}$) $r = \sin\theta - \cos\theta$

① NO CONCLUSION

Find all values of θ (for $0 \le \theta < 2\pi$) where the graph of $r = 3 + 6\sin\theta$ passes through the pole. **SCORE: ___ / 3 POINTS**

① $0 = 3 + 6\sin\theta$

($\frac{1}{2}$) $\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

① ($\frac{1}{2}$)

Convert the rectangular equation $xy = 8$ to polar form. Simplify your final answer using identities. **SCORE: ___ / 3 POINTS**

① $r\cos\theta \, r\sin\theta = 8$

($1\frac{1}{2}$) $r^2 = \frac{8}{\sin\theta\cos\theta}$

$= \frac{16}{2\sin\theta\cos\theta} = \frac{16}{\sin 2\theta} = 16\csc 2\theta$ ($\frac{1}{2}$)

Convert the polar equation $r^2 = \cos 2\theta$ to rectangular form. **SCORE: ___ / 4 POINTS**

Your final answer must NOT have radicals, but may use factored expressions.

② $r^2 = \cos^2\theta - \sin^2\theta$

① $r^4 = r^2\cos^2\theta - r^2\sin^2\theta$

① $(x^2+y^2)^2 = x^2 - y^2$

OR

$r^2 = 2\cos^2\theta - 1$ ②

$r^4 = 2r^2\cos^2\theta - r^2$ ①

$(x^2+y^2)^2 = 2x^2 - (x^2+y^2)$ ($\frac{1}{2}$)

$(x^2+y^2)^2 = x^2 - y^2$ ($\frac{1}{2}$)

Convert the rectangular co-ordinates $(-3, -\sqrt{3})$ to polar co-ordinates using $r > 0$ and $0 \le \theta < 2\pi$. **SCORE: ___ / 2 POINTS**

$r = \sqrt{(-3)^2 + (-\sqrt{3})^2}$

$r = \sqrt{12} = 2\sqrt{3}$ $(\frac{1}{2})$

$\theta = \pi + \tan^{-1} \frac{-\sqrt{3}}{-3}$

$= \pi + \tan^{-1} \frac{\sqrt{3}}{3} = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ $(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$

$\left(2\sqrt{3}, \frac{7\pi}{6}\right)$

NO POINTS BUT MINUS $\frac{1}{2}$ IF MISSING

Fill in the blanks. **SCORE: ___ / 5 POINTS**

[a] The asymptotes of a hyperbola intersect at the CENTER (1) of the hyperbola.

[b] If replacing (r, θ) in a polar equation with $(-r, \pi - \theta)$ yields an equivalent equation, then the graph of the equation is symmetric with respect to THE POLAR AXIS (1).

[c] If the point with rectangular co-ordinates $(0, -7)$ has polar co-ordinates $(7, \theta)$ and $0 \le \theta < 2\pi$, then $\theta = \frac{3\pi}{2}$ (1).

[d] In the polar co-ordinate system, the locus of points with co-ordinates $(0, \theta)$ is called THE POLE (1).

[e] The conic with equation $97x^2 - 97x + 109y^2 + 253y - 671 = 0$ is a/an ELLIPSE (1).

A hyperbola has asymptotes $2x - y - 8 = 0$ and $2x + y - 8 = 0$. If one of the foci is at $(-1, 0)$, find the equation of the hyperbola. **SCORE: ___ / 7 POINTS**

NOTE: $a = h,\ b = v$

$y = 2x - 8$

$y = -2x + 8$

$m = \pm 2$

$2x - 8 = -2x + 8$

$4x = 16$

$x = 4$

$y = 0$

CENTER $(4, 0)$ (1)

SEMI-FOCAL LENGTH $= 4 - -1 = 5$

HORIZONTAL TRANSVERSE AXIS

(1) $\frac{v}{h} = 2$

$v = 2h$

$v^2 + h^2 = 5^2$ (1)

$4h^2 + h^2 = 25$

$5h^2 = 25$ (1)

$h^2 = 5$

$h = \sqrt{5}$ $(\frac{1}{2})$

$v = 2\sqrt{5}$ $(\frac{1}{2})$

$$\frac{(x-4)^2}{5} - \frac{y^2}{20} = 1 \quad (2)$$

A point has polar co-ordinates $\left(6, \frac{8\pi}{7}\right)$. **SCORE: ___ / 2 POINTS**

[a] Find another polar representation for this point using $r > 0$ and $-2\pi < \theta < 2\pi$.

$\left(6, \frac{8\pi}{7} - 2\pi\right) = \left(6, -\frac{6\pi}{7}\right)$ (1)

[b] Find another polar representation for this point using $r < 0$ and $-2\pi < \theta < 2\pi$.

$\left(-6, \frac{8\pi}{7} + \pi\right) = \left(-6, \frac{15\pi}{7}\right)$ (1) OR $\left(-6, \frac{8\pi}{7} - \pi\right) = \left(-6, \frac{\pi}{7}\right)$ (1)