Fri Feb 10, 2012

SCORE: ___ / 30 POINTS

TO GET FULL CREDIT: YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER

Test $r = \cos\theta - \sin\theta$ for symmetry with respect to $\theta = \frac{\pi}{2}$. State clearly the conclusion of the test. **SCORE: ___ / 4 POINTS**

1

$-r = \cos(-\theta) - \sin(-\theta)$

$-r = \cos\theta + \sin\theta$

$r = -\cos\theta - \sin\theta$

$\frac{1}{2}$

$r = \cos(\pi-\theta) - \sin(\pi-\theta)$ 1

$r = \cos\pi\cos\theta + \sin\pi\sin\theta$
$- [\sin\pi\cos\theta - \cos\pi\sin\theta]$

$r = -\cos\theta - \sin\theta$ $\frac{1}{2}$

NO CONCLUSION 1

Find all values of θ (for $0 \le \theta < 2\pi$) where the graph of $r = 3 + 6\cos\theta$ passes through the pole. **SCORE: ___ / 3 POINTS**

1 $0 = 3 + 6\cos\theta$

$\cos\theta = -\frac{1}{2}$

$\frac{1}{2}$ $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

1 $\frac{1}{2}$

Convert the rectangular equation $x^2 - y^2 = 6$ to polar form. Simplify your final answer using identities. **SCORE: ___ / 3 POINTS**

$r^2\cos^2\theta - r^2\sin^2\theta = 6$

1

$r^2 = \frac{6}{\cos^2\theta - \sin^2\theta} = \frac{6}{\cos 2\theta} = 6\sec 2\theta$

$1\frac{1}{2}$ $\frac{1}{2}$

Convert the polar equation $r^2 = \csc 2\theta$ to rectangular form.
Your final answer must NOT have radicals, but may use factored expressions. **SCORE: ___ / 4 POINTS**

$r^2 = \frac{1}{\sin 2\theta}$

$r^2 = \frac{1}{2\sin\theta\cos\theta}$ 2

$2r^2\sin\theta\cos\theta = 1$ 1

$2(r\sin\theta)(r\cos\theta) = 1$

1

$2xy = 1$ OR $y = \frac{1}{2x}$

Convert the rectangular co-ordinates $(-\sqrt{3}, 3)$ to polar co-ordinates using $r > 0$ and $0 \le \theta < 2\pi$. **SCORE: ___ / 2 POINTS**

$r = \sqrt{(-\sqrt{3})^2 + 3^2}$ $\qquad \theta = \pi + \tan^{-1}\frac{3}{-\sqrt{3}}$ NO POINTS BUT MINUS $\frac{1}{2}$ IF MISSING $\rightarrow (2\sqrt{3}, \frac{2\pi}{3})$

$r = \sqrt{12} = \underline{2\sqrt{3}}$ $\frac{1}{2}$ $\qquad \theta = \pi + \tan^{-1}(-\sqrt{3}) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

Fill in the blanks. **SCORE: ___ / 5 POINTS**

[a] The asymptotes of a hyperbola intersect at the CENTER (1) of the hyperbola.

[b] If replacing (r, θ) in a polar equation with $(-r, -\theta)$ yields an equivalent equation, then the graph of the equation is symmetric with respect to $\theta = \frac{\pi}{2}$ (1).

[c] If the point with rectangular co-ordinates $(0, -7)$ has polar co-ordinates $(7, \theta)$ and $0 \le \theta < 2\pi$, then $\theta =$ $\frac{3\pi}{2}$ (1).

[d] In the polar co-ordinate system, the locus of points with co-ordinates $(0, \theta)$ is called THE POLE (1).

[e] The conic with equation $109x^2 + 109x + 97y^2 + 253y - 671 = 0$ is a/an ELLIPSE (1).

A hyperbola has asymptotes $3x - y + 6 = 0$ and $3x + y + 6 = 0$. If one of the foci is at $(8, 0)$, find the equation of the hyperbola. **SCORE: ___ / 7 POINTS**

NOTE: $a = h$, $b = v$

$y = 3x + 6$
$y = -3x - 6$ $\qquad m = \pm 3$
$3x + 6 = -3x - 6$
$6x = -12$
$x = -2$
$y = 0$
CENTER $(-2, 0)$ (1)
SEMI-FOCAL LENGTH $= 8 - -2 = 10$
HORIZONTAL TRANSVERSE AXIS

(1) $\frac{v}{h} = 3$ $\qquad v^2 + h^2 = 10^2$ (1)
$v = 3h$ $\qquad 9h^2 + h^2 = 100$
$10h^2 = 100$ (1)
$h^2 = 10$
$h = \sqrt{10}$ $\frac{1}{2}$
$v = 3\sqrt{10}$ $\frac{1}{2}$

$$\frac{(x+2)^2}{10} - \frac{y^2}{90} = 1$$ (2)

A point has polar co-ordinates $\left(5, \frac{6\pi}{7}\right)$. **SCORE: ___ / 2 POINTS**

[a] Find another polar representation for this point using $r > 0$ and $-2\pi < \theta < 2\pi$.

$(5, \frac{6\pi}{7} - 2\pi) = (5, -\frac{8\pi}{7})$ (1)

[b] Find another polar representation for this point using $r < 0$ and $-2\pi < \theta < 2\pi$.

$(-5, \frac{6\pi}{7} + \pi) = (-5, \frac{13\pi}{7})$ (1) OR $(-5, \frac{6\pi}{7} - \pi) = (-5, -\frac{\pi}{7})$ (1)