Fri Feb 10, 2012

SCORE: ___ / 30 POINTS

TO GET FULL CREDIT: YOU MUST SHOW THE WORK THAT LEAD TO YOUR ANSWER

Test $r = \cos\theta - \sin\theta$ for symmetry with respect to the polar axis. State clearly the conclusion of the test. **SCORE: ___ / 4 POINTS**

1 $r = \cos(-\theta) - \sin(-\theta)$

$r = \cos\theta + \sin\theta$

$\frac{1}{2}$

$-r = \cos(\pi - \theta) - \sin(\pi - \theta)$ 1

$-r = \cos\pi\cos\theta + \sin\pi\sin\theta - [\sin\pi\cos\theta - \cos\pi\sin\theta]$

$-r = -\cos\theta - \sin\theta$

$\frac{1}{2}$ $r = \cos\theta + \sin\theta$ 1 NO CONCLUSION

Find all values of θ (for $0 \le \theta < 2\pi$) where the graph of $r = 3 + 6\cos\theta$ passes through the pole. **SCORE: ___ / 3 POINTS**

1 $0 = 3 + 6\cos\theta$

$\cos\theta = -\frac{1}{2}$

$\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

1 $\frac{1}{2}$

Convert the rectangular equation $x^2 - y^2 = 8$ to polar form. Simplify your final answer using identities. **SCORE: ___ / 3 POINTS**

1 $r^2\cos^2\theta - r^2\sin^2\theta = 8$

$r^2 = \frac{8}{\cos^2\theta - \sin^2\theta} = \frac{8}{\cos 2\theta} = 8\sec 2\theta$

$1\frac{1}{2}$ $\frac{1}{2}$

Convert the polar equation $r^2 = \csc 2\theta$ to rectangular form. **SCORE: ___ / 4 POINTS**

Your final answer must NOT have radicals, but may use factored expressions.

$r^2 = \frac{1}{\sin 2\theta}$

$r^2 = \frac{1}{2\sin\theta\cos\theta}$ 2

$2r^2\sin\theta\cos\theta = 1$ 1

$2(r\sin\theta)(r\cos\theta) = 1$

1 $2xy = 1$ OR $y = \frac{1}{2x}$

Convert the rectangular co-ordinates $(-3, \sqrt{3})$ to polar co-ordinates using $r > 0$ and $0 \le \theta < 2\pi$. **SCORE: ___ / 2 POINTS**

$r = \sqrt{(-3)^2 + (\sqrt{3})^2}$ $\quad \theta = \pi + \tan^{-1}\frac{\sqrt{3}}{-3}$ ($\frac{1}{2}$)

$r = \sqrt{12} = 2\sqrt{3}$ ($\frac{1}{2}$) $\quad \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ ($\frac{1}{2}$) ($\frac{1}{2}$)

$\left(2\sqrt{3}, \frac{5\pi}{6}\right)$ NO POINTS, BUT MINUS $\frac{1}{2}$ IF MISSING

Fill in the blanks. **SCORE: ___ / 5 POINTS**

[a] The asymptotes of a hyperbola intersect at the CENTER (1) of the hyperbola.

[b] If replacing (r, θ) in a polar equation with $(-r, -\theta)$ yields an equivalent equation, then the graph of the equation is symmetric with respect to $\theta = \frac{\pi}{2}$ (1).

[c] If the point with rectangular co-ordinates $(0, -7)$ has polar co-ordinates $(7, \theta)$ and $0 \le \theta < 2\pi$, then $\theta = \frac{3\pi}{2}$ (1).

[d] In the polar co-ordinate system, the locus of points with co-ordinates $(0, \theta)$ is called THE POLE (1).

[e] The conic with equation $97x^2 - 97x + 253y - 671 = 0$ is a/an PARABOLA (1).

A hyperbola has asymptotes $3x - y + 9 = 0$ and $3x + y + 9 = 0$. If one of the foci is at $(7, 0)$, find the equation of the hyperbola. **SCORE: ___ / 7 POINTS**

NOTE: $a = h$, $b = v$

$y = 3x + 9$ $\quad m = \pm 3$

$y = -3x - 9$

$3x + 9 = -3x - 9$

$6x = -18$

$x = -3$

$y = 0$

CENTER $(-3, 0)$ (1)

SEMI-FOCAL LENGTH $= 7 - -3 = 10$

HORIZONTAL TRANSVERSE AXIS

(1) $\frac{v}{h} = 3$ $\quad v^2 + h^2 = 10^2$ (1)

$v = 3h$ $\quad 9h^2 + h^2 = 100$

$10h^2 = 100$ (1)

$h^2 = 10$

$h = \sqrt{10}$ ($\frac{1}{2}$)

$v = 3\sqrt{10}$ ($\frac{1}{2}$)

$$\frac{(x+3)^2}{10} - \frac{y^2}{90} = 1 \quad (2)$$

A point has polar co-ordinates $\left(4, \frac{6\pi}{5}\right)$. **SCORE: ___ / 2 POINTS**

[a] Find another polar representation for this point using $r > 0$ and $-2\pi < \theta < 2\pi$.

$\left(4, \frac{6\pi}{5} - 2\pi\right) = \left(4, \frac{-4\pi}{5}\right)$ (1)

[b] Find another polar representation for this point using $r < 0$ and $-2\pi < \theta < 2\pi$.

$\left(-4, \frac{6\pi}{5} + \pi\right) = \left(-4, \frac{11\pi}{5}\right)$ (1) OR $\left(-4, \frac{6\pi}{5} - \pi\right) = \left(-4, \frac{\pi}{5}\right)$ (1)