Math 43 (8:30am Class)
Quiz 6 Version X
Fri Mar 2, 2012

NAME YOU ASKED TO BE CALLED IN CLASS: _____

SCORE: ___ / 30 POINTS

TO GET FULL CREDIT: YOU MUST SHOW THE WORK THAT LEADS TO YOUR ANSWER

If $\mathbf{y} = -5\mathbf{i} - 3\mathbf{j}$ and $\mathbf{z} = -7\mathbf{i} + 12\mathbf{j}$, determine if the angle between $\mathbf{y}$ and $\mathbf{z}$ is right, obtuse or acute. **SCORE: ___ / 2 POINTS**

$y \cdot z = -1 < 0$ OBTUSE

(1) (1) ← MUST HAVE BOTH REASON + "OBTUSE"

If $\mathbf{q} = \langle 3, -2 \rangle$ and $\mathbf{r} = \langle -4, 7 \rangle$, write $\mathbf{r}$ as the sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{q}}\,\mathbf{r}$. **SCORE: ___ / 5 POINTS**

$\text{PROJ}_{\vec{q}}\,\vec{r} = \frac{\vec{q}\cdot\vec{r}}{\vec{q}\cdot\vec{q}}\vec{q}$

$= \frac{-26}{13}\langle 3, -2 \rangle$ (1) (1)

$= -2\langle 3, -2 \rangle$

$= \langle -6, 4 \rangle$ (1)

$\vec{r} - \text{PROJ}_{\vec{q}}\,\vec{r} = \langle -4, 7 \rangle - \langle -6, 4 \rangle$

$= \langle 2, 3 \rangle$ ($1\frac{1}{2}$)

$\vec{r} = \langle -6, 4 \rangle + \langle 2, 3 \rangle$ ($\frac{1}{2}$)

The force given by the vector $\langle 7, 3 \rangle$ moves an object from the point $(-1, -6)$ to the point $(-4, 2)$. Find the work done. **SCORE: ___ / 3 POINTS**

$\langle 7, 3 \rangle \cdot \langle -4 - -1, 2 - -6 \rangle = \langle 7, 3 \rangle \cdot \langle -3, 8 \rangle = 3$

(1) (1) (1)

A point is 11 units above the $xy-$plane, 25 units behind the $yz-$plane, and lies in the $xz-$plane. Find its co-ordinates. **SCORE: ___ / 2 POINTS**

$(-25, 0, 11)$

2 POINTS IF ALL 3 CO-ORDINATES RIGHT

1 POINT IF 2 OUT OF 3 RIGHT

0 POINTS IF 0 OR 1 RIGHT

Find the octant in which $(-2, 6, -3)$ is located. **SCORE: ___ / 2 POINTS**

$x < 0$, $y > 0$ } Q_2 $\quad z < 0 \rightarrow O_{2+4} = O_6$ (2)

A diameter of a sphere has endpoints $(1, -5, 2)$ and $(-2, -7, -4)$. **SCORE: ___ / 4 POINTS**

Find the standard form of the equation of the sphere.

CENTER = MIDPOINT = $\left(-\frac{1}{2}, -6, -1\right)$ (1)

DIAMETER = $\sqrt{(-3)^2 + (-2)^2 + (-6)^2} = \sqrt{49} = 7 \longrightarrow$ RADIUS = $\frac{7}{2}$ ($1\frac{1}{2}$)

$\left(x + \frac{1}{2}\right)^2 + (y+6)^2 + (z+1)^2 = \frac{49}{4}$

($1\frac{1}{2}$)

FILL IN THE BLANK: If θ is the angle between vectors **p** and **q**, **SCORE: ___ / 1 POINT**

then $\| \mathbf{p} \| \| \mathbf{q} \| \sin\theta =$ $\|\vec{p} \times \vec{q}\|$. (1)

ONLY ½ POINT IF YOU SAID $\vec{p} \times \vec{q}$ WITHOUT $\| \ \|$

Let $\mathbf{u} = \langle 2, 3, -1 \rangle$ and $\mathbf{v} = \langle a, -6, -2 \rangle$. **SCORE: ___ / 5 POINTS**

[a] Is there a value of a such that **u** and **v** are orthogonal ? If so, find it. If not, show why no such value exists.

$\vec{u} \cdot \vec{v} = 0$

$2a - 18 + 2 = 0$ (1)

$a = 8$ (1)

[b] Is there a value of a such that **u** and **v** are parallel ? If so, find it. If not, show why no such value exists.

$\vec{u} = k\vec{v}$

$\langle 2, 3, -1 \rangle = k\langle a, -6, -2 \rangle$

$\langle 2, 3, -1 \rangle = \langle ka, -6k, -2k \rangle$ (1)

$2 = ka$

$3 = -6k \longrightarrow k = -\frac{1}{2}$

$-1 = -2k \longrightarrow k = \frac{1}{2}$

($\frac{1}{2}$) ($\frac{1}{2}$)

IMPOSSIBLE

NO SUCH a (1)

Let $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{f} = -\mathbf{j} - 2\mathbf{k}$. **SCORE: ___ / 6 POINTS**

[a] Find $\mathbf{b} \times \mathbf{f}$.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 1 \\ 0 & -1 & -2 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 3 & -2 \\ 0 & -1 \end{matrix} = (4\vec{i} - 3\vec{k}) - (-\vec{i} - 6\vec{j}) = 5\vec{i} + 6\vec{j} - 3\vec{k}$$

($1\frac{1}{2}$) ($1\frac{1}{2}$) ($1\frac{1}{2}$)

CAN ALSO USE COMPONENT FORM

[b] Find a unit vector that is orthogonal to both **b** and **f**.

$\frac{1}{\|\vec{b} \times \vec{f}\|}(\vec{b} \times \vec{f}) = \frac{1}{\sqrt{70}}\langle 5, 6, -3 \rangle = \left\langle \frac{5}{\sqrt{70}}, \frac{6}{\sqrt{70}}, \frac{-3}{\sqrt{70}} \right\rangle$

(1) ($\frac{1}{2}$)