Math 43 (9:30am Class)
Quiz 6 Version Y
Fri Mar 2, 2012

NAME YOU ASKED TO BE CALLED IN CLASS: ____________________

SCORE: ___ / 30 POINTS

TO GET FULL CREDIT: YOU MUST SHOW THE WORK THAT LEADS TO YOUR ANSWER

If $\mathbf{y} = -2\,\mathbf{i} - 7\,\mathbf{j}$ and $\mathbf{z} = -10\,\mathbf{i} + 3\,\mathbf{j}$, determine if the angle between $\mathbf{y}$ and $\mathbf{z}$ is right, obtuse or acute. **SCORE: ___ / 2 POINTS**

$y \cdot z = \underbrace{-1}_{1} \underbrace{< 0 \text{ OBTUSE}}_{1}$ ← MUST HAVE BOTH REASON & "OBTUSE"

If $\mathbf{q} = \langle 4, -1 \rangle$ and $\mathbf{r} = \langle -5, 14 \rangle$, write $\mathbf{r}$ as the sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{q}}\,\mathbf{r}$. **SCORE: ___ / 5 POINTS**

$$\text{PROJ}_{\vec{q}}\vec{r} = \frac{\vec{q}\cdot\vec{r}}{\vec{q}\cdot\vec{q}}\vec{q}$$

$$= \frac{\overset{1}{\boxed{-34}}}{\underset{1}{\boxed{17}}}\langle 4,-1\rangle$$

$$= -2\langle 4,-1\rangle$$

$$= \underbrace{\langle -8, 2\rangle}_{1}$$

$$\vec{r} - \text{PROJ}_{\vec{q}}\vec{r} = \langle -5, 14\rangle - \langle -8, 2\rangle$$

$$= \underbrace{\langle 3, 12\rangle}_{1\frac{1}{2}}$$

$$\vec{r} = \underbrace{\langle -8, 2\rangle + \langle 3, 12\rangle}_{\frac{1}{2}}$$

The force given by the vector $\langle 3, 7 \rangle$ moves an object from the point $(-1, -3)$ to the point $(-5, 2)$. **SCORE: ___ / 3 POINTS**
Find the work done.

$$\langle 3,7\rangle \cdot \langle -5-{}^{-}1, 2-{}^{-}3\rangle = \overbrace{\langle 3,7\rangle \cdot \underbrace{\langle -4,5\rangle}_{1}}^{1} = \underbrace{23}_{1}$$

A point is 11 units to the left of the xz-plane, 25 units above the xy-plane, and lies in the yz-plane. **SCORE: ___ / 2 POINTS**
Find its co-ordinates.

$(0, -11, 25)$

2 POINTS IF ALL 3 CO-ORDINATES RIGHT
1 POINTS IF 2 OUT OF 3 " RIGHT
0 POINTS IF 0 OR 1 " RIGHT

Find the octant in which $(2, -6, -3)$ is located. **SCORE: ___ / 2 POINTS**

$$\left.\begin{matrix} x > 0 \\ y < 0 \end{matrix}\right\} Q_4 \qquad z < 0 \rightarrow O_{4+4} = \underbrace{O_8}_{2}$$

A diameter of a sphere has endpoints $(-2, -1, 5)$ and $(4, 2, 7)$. **SCORE: ___ / 4 POINTS**

Find the standard form of the equation of the sphere.

CENTER = MIDPOINT = $(1, \frac{1}{2}, 6)$ (1)

DIAMETER = $\sqrt{6^2+3^2+2^2} = \sqrt{49} = 7 \longrightarrow$ RADIUS = $\frac{7}{2}$ ($1\frac{1}{2}$)

$(x-1)^2+(y-\frac{1}{2})^2+(z-6)^2 = \frac{49}{4}$ ($1\frac{1}{2}$)

FILL IN THE BLANK: If θ is the angle between vectors **p** and **q**, **SCORE: ___ / 1 POINT**

then $\|\mathbf{p}\|\,\|\mathbf{q}\|\sin\theta$ = $\|\vec{p}\times\vec{q}\|$. (1)

ONLY $\frac{1}{2}$ POINT IF YOU SAID $\vec{p}\times\vec{q}$ WITHOUT $\|\ \ \|$

Let $\mathbf{u} = \langle 2, -4, 3\rangle$ and $\mathbf{v} = \langle -6, -12, a\rangle$. **SCORE: ___ / 5 POINTS**

[a] Is there a value of a such that **u** and **v** are orthogonal ? If so, find it. If not, show why no such value exists.

$\vec{u}\cdot\vec{v} = 0$

$-12+48+3a=0$ (1)

$a=-12$ (1)

[b] Is there a value of a such that **u** and **v** are parallel ? If so, find it. If not, show why no such value exists.

$\vec{u} = k\vec{v}$

$\langle 2,-4,3\rangle = k\langle -6,-12,a\rangle$

$\langle 2,-4,3\rangle = \langle -6k,-12k,ka\rangle$ (1)

($\frac{1}{2}$) $2=-6k \rightarrow k=-\frac{1}{3}$

$-4=-12k \rightarrow k=\frac{1}{3}$ } IMPOSSIBLE ($\frac{1}{2}$)

$3=ka$

NO SUCH a (1)

Let $\mathbf{b} = 2\,\mathbf{i} - 3\,\mathbf{j} + \mathbf{k}$ and $\mathbf{f} = 3\,\mathbf{j} - 2\,\mathbf{k}$. **SCORE: ___ / 6 POINTS**

[a] Find $\mathbf{b}\times\mathbf{f}$.

$$\left|\begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 1 \\ 0 & 3 & -2 \end{array}\right|\begin{array}{cc} \vec{i} & \vec{j} \\ 2 & -3 \\ 0 & 3 \end{array} = (6\vec{i}+6\vec{k})-(3\vec{i}-4\vec{j}) = \underbrace{3\vec{i}}_{1\frac{1}{2}}+\underbrace{4\vec{j}}_{1\frac{1}{2}}+\underbrace{6\vec{k}}_{1\frac{1}{2}}$$

CAN ALSO BE IN COMPONENT FORM

[b] Find a unit vector that is orthogonal to both **b** and **f**.

$$\frac{1}{\|\vec{b}\times\vec{f}\|}(\vec{b}\times\vec{f}) = \underbrace{\frac{1}{\sqrt{61}}\langle 3,4,6\rangle}_{1} = \underbrace{\left\langle \frac{3}{\sqrt{61}}, \frac{4}{\sqrt{61}}, \frac{6}{\sqrt{61}}\right\rangle}_{\frac{1}{2}}$$