SHOW THE ROW OPERATIONS USED FOR EACH STEP OF GAUSS-JORDAN ELIMINATION (AS IN LECTURE)

Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \\ -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 \\ -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & -1 & 3 \\ 1 & -2 & 2 \end{bmatrix}$.

SCORE: ___ / 9 POINTS

[a] Which **one** of the products BA, CA or CB exists ? Find its value.

$$CA = \begin{bmatrix} 0 & -1 & 3 \\ 1 & -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 0-1-6 & 0-3+3 \\ 2-2-4 & -1-6+2 \end{bmatrix} = \begin{bmatrix} -7 & 0 \\ -4 & -5 \end{bmatrix}$$

[b] Which **one** of the products A^2, B^2 or C^2 exists ? Find its value.

$$B^2 = \begin{bmatrix} -2 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 4-1 & -2+2 \\ 2-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

[c] If $D = \begin{bmatrix} 2 & 3 & 0 \\ -3 & 1 & -1 \end{bmatrix}$ and $4C - 2X = 3D$, find the value of X.

$$X = \frac{1}{2}(4C-3D) = \frac{1}{2}\left(\begin{bmatrix} 0 & -4 & 12 \\ 4 & -8 & 8 \end{bmatrix} - \begin{bmatrix} 6 & 9 & 0 \\ -9 & 3 & -3 \end{bmatrix}\right) = \begin{bmatrix} -3 & \frac{-13}{2} & 6 \\ \frac{13}{2} & -\frac{11}{2} & \frac{11}{2} \end{bmatrix}$$

Consider the system of equations
$$\begin{aligned} 2x - 5y - 31z &= 8 \\ -x + y + 8z &= -1 \end{aligned}$$

SCORE: ___ / 7 POINTS

[a] Write the system as a matrix equation of the form $AX = B$.

$$\begin{bmatrix} 2 & -5 & -31 \\ -1 & 1 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

[b] Write the augmented matrix for this system. (The equations must be in the order listed above.)

$$\left[\begin{array}{ccc|c} 2 & -5 & -31 & 8 \\ -1 & 1 & 8 & -1 \end{array}\right] R_1 \leftrightarrow R_2$$

[c] Solve the system using **Gauss-Jordan elimination as shown in lecture**. Write your final answer in an appropriate form.

$$\left[\begin{array}{ccc|c} -1 & 1 & 8 & -1 \\ 2 & -5 & -31 & 8 \end{array}\right] (-1)R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -8 & 1 \\ 2 & -5 & -31 & 8 \end{array}\right] R_2 + (-2)R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -8 & 1 \\ 0 & -3 & -15 & 6 \end{array}\right] -\frac{1}{3}R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -8 & 1 \\ 0 & 1 & 5 & -2 \end{array}\right] R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -1 \\ 0 & 1 & 5 & -2 \end{array}\right]$$

$x - 3z = -1$
$y + 5z = -2$

$x = 3z - 1$
$y = -5z - 2$

$x = 3t - 1$
$y = -5t - 2$
$z = t$

Let $A = \begin{bmatrix} 7 & 8 & -6 & 2 \\ 5 & -3 & 0 & 11 \\ -4 & -9 & 10 & -1 \end{bmatrix}$. Fill in the blanks below. **SCORE: ___ / 5 POINTS**

[a] The order of A is $\underline{3 \times 4}$.

[b] $a_{32} = \underline{-9}$.

[c] If $a_{ij} = 11$, then $i = \underline{2}$ and $j = \underline{4}$.

[d] If $A = BC$ and B has 2 columns, then the order of B is $\underline{3 \times 2}$ and the order of C is $\underline{2 \times 4}$.

Mudd Coffee sells 3 mixes of coffee beans. **SCORE: ___ / 9 POINTS**
Bitter As Bile is 1 part Australian beans, 2 parts Brazilian beans and 3 parts Indonesian beans.
Darker Than Dirt is 1 part Australian beans and 1 part Indonesian beans.
Sludgy Like Soot is 2 parts Brazilian beans and 1 part Indonesian beans.
You would like to buy exactly 2 pounds of Australian beans, 1 pound of Brazilian beans and 2 pounds of Indonesian beans by combining amounts of the 3 mixes.

[a] Write an augmented matrix for this problem. **SCALE THE EQUATIONS SO ALL MATRIX ENTRIES ARE INTEGERS.**

You may ask for the system of equations. Your maximum score for this entire question will then be 6 points (not 9 points).

b = # POUNDS OF BAB
d = DTD
s = SLS

$\frac{1}{6}b + \frac{1}{2}d = 2$ POUNDS AUSTRALIAN $\quad b + 3d = 12$

$\frac{1}{3}b + \frac{2}{3}s = 1$ BRAZILIAN $\quad b + 2s = 3$

$\frac{1}{2}b + \frac{1}{2}d + \frac{1}{3}s = 2$ INDONESIAN $\quad 3b + 3d + 2s = 12$

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 12 \\ 1 & 0 & 2 & 3 \\ 3 & 3 & 2 & 12 \end{array}\right] \begin{array}{l} \\ R_2 + (-1)R_1 \\ R_3 + (-3)R_1 \end{array}$$

[b] Use **Gauss-Jordan elimination as shown in lecture** to find the reduced row echelon form of the augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 12 \\ 0 & -3 & 2 & -9 \\ 0 & -6 & 2 & -24 \end{array}\right] \begin{array}{l} \\ -\frac{1}{3}R_2 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 12 \\ 0 & 1 & -2/3 & 3 \\ 0 & -6 & 2 & -24 \end{array}\right] \begin{array}{l} \\ \\ R_3 + 6R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 12 \\ 0 & 1 & -2/3 & 3 \\ 0 & 0 & -2 & -6 \end{array}\right] \begin{array}{l} \\ \\ -\frac{1}{2}R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 12 \\ 0 & 1 & -2/3 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} \\ R_2 + \frac{2}{3}R_3 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 12 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} R_1 + (-3)R_2 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} b = -3 \\ d = 5 \\ s = 3 \end{array}$$

CHECK: $\frac{1}{6}(-3) + \frac{1}{2}(5) = -\frac{1}{2} + \frac{5}{2} = 2$ ✓

$\frac{1}{3}(-3) + \frac{2}{3}(3) = -1 + 2 = 1$ ✓

$\frac{1}{2}(-3) + \frac{1}{2}(5) + \frac{1}{3}(3) = -\frac{3}{2} + \frac{5}{2} + 1 = 2$ ✓

[c] Solve the original problem. **Summarize your answer in a sentence.**

YOU CAN'T BUY THAT EXACT COMBINATION SINCE IT REQUIRES A NEGATIVE AMOUNT OF BITTER AS BILE