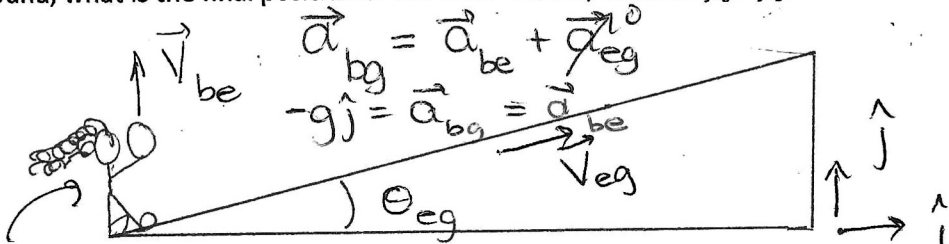


Show all your work for full credit. No calculators, electronic devices, scratch papers or note cards are allowed. Please box your final answer.

1. A physics student, hoping to earn extra credit, conducts an experiment while riding an escalator at the mall. The student tosses a ball straight up while riding the escalator and catches it. The escalator makes an angle θ_{eg} with the horizontal and has constant speed V_{eg} . The student on the escalator throws the ball with speed V_{be} , at position $X_{ibe} = X_{ieg} = 0, Y_{ibe} = Y_{ieg} = 0$ as shown. Someone on the ground observes the experiment. According to the person on the ground, what is the final position of the ball? That is, what is X_{fbg}, Y_{fbg} ?



$$\vec{v}_{ibg} = \vec{v}_{ibe} + \vec{v}_{ieg}$$

$$v_{ibgx} = v_{ibex} + v_{iegx}$$

$$v_{ibgx} = v_{eg} \cos \theta_{eg}$$

$$v_{ibgy} = v_{ibey} + v_{iegy}$$

$$v_{ibgy} = v_{be} + v_{eg} \sin \theta_{eg}$$

(X_{ibe}, Y_{ibe})

WHAT DO I SEE?

$$X_{fbg} = X_{ibg} + v_{ibg} t + \frac{1}{2} a_{ibg} t^2$$

$$X_{fbg} = v_{eg} \cos \theta_{eg} t$$

$$Y_{fbg} = Y_{ibg} + v_{ibg} t + \frac{1}{2} a_{ibgy} t^2$$

$$Y_{fbg} = (v_{be} + v_{eg} \sin \theta_{eg}) t + \frac{1}{2} (-g) t^2$$

NEED t ! THE TIME OF FLIGHT IS FOUND IN THE ESCALATOR FRAME.

$$Y_{fbe} = Y_{ibe} + v_{ibe} t + \frac{1}{2} a_{ibe} t^2$$

$$0 = v_{be} t - \frac{1}{2} g t^2$$

$$t = \frac{2v_{be}}{g} \quad \text{use in * and **}$$

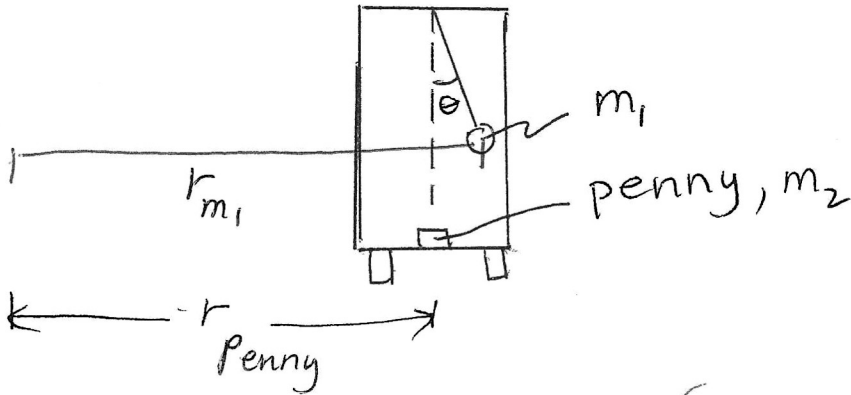
$$X_{fbg} = \frac{2v_{be} v_{eg} \cos \theta_{eg}}{g}$$

$$Y_{fbg} = (v_{be} + v_{eg} \sin \theta_{eg}) \frac{2v_{be}}{g} - \frac{1}{2} g \left(\frac{2v_{be}}{g} \right)^2$$

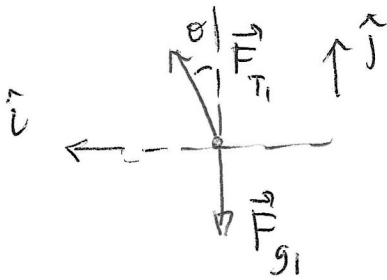
$$Y_{fbg} = \frac{2v_{be}^2}{g} + \frac{2v_{be} v_{eg} \sin \theta_{eg}}{g} - \frac{2v_{be}^2}{g}$$

$$Y_{fbg} = \frac{2v_{be} v_{eg} \sin \theta_{eg}}{g}$$

2. A truck is driving on a flat, horizontal circular track of radius r . Inside the truck compartment, a pendulum (a point particle of mass m_1 attached to a string of length L) is hanging from the ceiling and makes an angle θ with the vertical as shown. On the floor of the truck, there is a penny of mass m_2 which doesn't slide as the truck circles the track. What is the minimum coefficient of static friction, μ_s between the penny and the truck floor? Your full-credit answer will be in terms of θ ?



SYSTEM: pendulum m_1



$$\vec{F}_{\text{net},1} = m_1 \vec{a}_1$$

$$\vec{F}_{T1} + \vec{F}_{g1} = m_1 \vec{a}_1$$

$$\hat{i}: F_T \sin \theta = m_1 a_c \quad *$$

$$\hat{j}: F_T \cos \theta - F_{g1} = 0$$

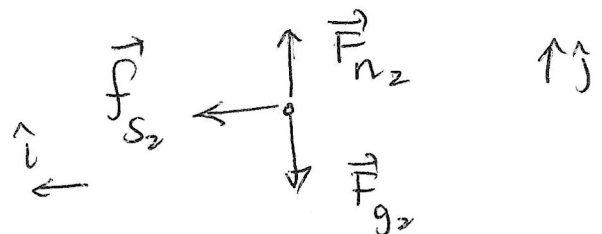
$$F_T = \frac{m_1 g}{\cos \theta} \quad **$$

combine * and **

$$\frac{m_1 g \sin \theta}{\cos \theta} = m_1 a_c$$

$$a_c = g \tan \theta \quad ***$$

SYSTEM: PENNY m_2



$$\vec{F}_{\text{net},2} = m_2 \vec{a}_2$$

$$\vec{F}_{n2} + \vec{F}_{g2} + \vec{f}_{s2} = m_2 \vec{a}_2$$

$$\hat{i}: f_{s2} = m_2 a_c$$

$$\hat{j}: F_{n2} - F_{g2} = 0$$

$$F_{n2} = m_2 g$$

$$f_s \leq \mu_s F_n$$

$$f_s \leq \mu_s m_2 g$$

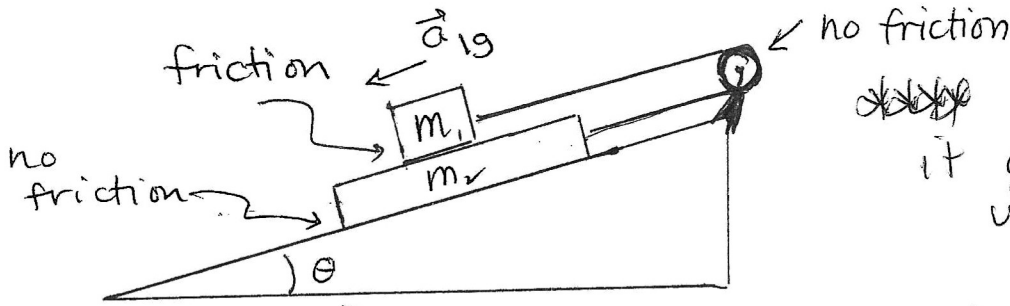
$$m_2 a_c \leq \mu_s m_2 g \quad \text{use ***}$$

$$g \tan \theta \leq \mu_s g$$

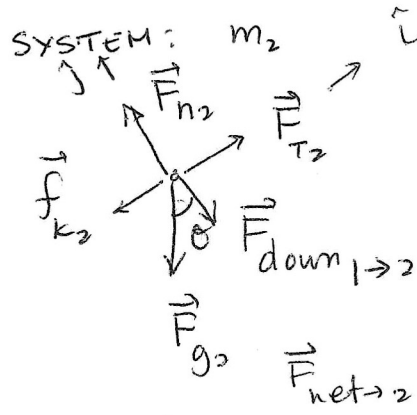
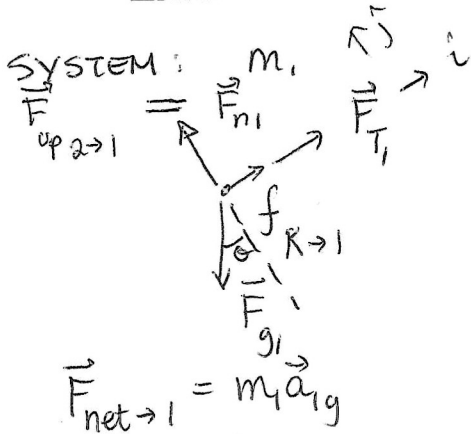
$$\mu_s \geq \tan \theta$$

$$\boxed{\mu_{s \text{ min}} = \tan \theta}$$

3. Mass m_1 is on top of m_2 . Both masses are on an incline of angle θ and they are connected by a string which passes around a pulley at the top of the incline as shown. The coefficient of kinetic friction between m_1 and m_2 is μ_k , and between m_2 and the incline is frictionless. Assume that the acceleration of m_1 is DOWN the incline (for now). Find \vec{a}_{1g} , the acceleration of m_1 with respect to the ground in terms of μ_k , m_1 , m_2 , θ , and g . By inspecting your answer for the acceleration of m_1 , determine what would be required for m_1 to accelerate UP the incline.



~~if~~ if $m_2 > m_1$
it goes up the incline
but
 $(m_1 - m_2) \sin \theta - 2\mu_k m_1 \cos \theta$
must be < 0
for it to go up.



$$\vec{F}_{T1} + \vec{F}_{g1} + \vec{F}_{N1} + \vec{f}_{k1} = m_1 \vec{a}_{1g}$$

$$\vec{F}_{g2} + \vec{F}_{T2} + \vec{F}_{N2} + \vec{F}_{d1 \rightarrow 2} + \vec{f}_{k2} = m_2 \vec{a}_{2g}$$

$$\hat{i}: F_{T1} + f_{k \rightarrow 1} - m_1 g \sin \theta = -m_1 a_{1g}$$

$$\hat{i}: F_{T2} - f_{k2} - m_2 g \sin \theta = m_2 a_{2g}$$

$$\hat{j}: F_{N1} - m_1 g \cos \theta = 0$$

$$\hat{j}: F_{N2} - F_{d \rightarrow 2} - m_2 g \cos \theta = 0$$

$$F_{N1} = m_1 g \cos \theta$$

$$f_{k1} = f_{k2} \quad \text{N3L}$$

$$f_{k \rightarrow 1} = \mu_k m_1 g \cos \theta = \mu_k F_{N1}$$

$$F_{T1} = -m_1 a_{1g} + m_1 g \sin \theta - \mu_k m_1 g \cos \theta$$

$$|\vec{F}_{T1}| = |\vec{F}_{T2}| = F_T$$

$$\text{set them equal } |\vec{a}_{1g}| = |\vec{a}_{2g}| = a$$

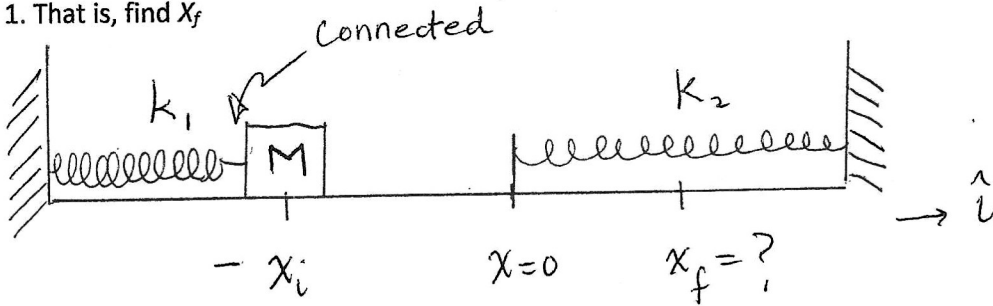
$$F_T = m_2 a + m_2 g \sin \theta + f_{k2}$$

$$-m_1 a + m_1 g \sin \theta - \mu_k m_1 g \cos \theta = m_2 a + m_2 g \sin \theta + \mu_k m_1 g \cos \theta$$

$$(m_2 + m_1) a = (m_1 - m_2) g \sin \theta - 2\mu_k m_1 g \cos \theta$$

$$\vec{a}_{1g} = g \left[\frac{(m_1 - m_2) \sin \theta - 2\mu_k m_1 \cos \theta}{m_1 + m_2} \right] (-\hat{i})$$

4. A mass M is connected to a spring of constant k_1 and initially compressing it by an amount X_i from the equilibrium position. The mass is allowed to move and at the equilibrium position the mass encounters (meets) a second spring of spring constant k_2 . The mass, still in contact with the first spring, now begins to compress the second spring. This motion is in one dimension only. Find the position where the mass will next be at rest compressing spring 2 and stretching spring 1. That is, find X_f



SYSTEM: MASS

$$W_{\text{net, ext}} = \Delta K$$

$$W_{S_1} + W_{S_2} = K_f - K_i \quad \left\{ \begin{array}{l} W_g = 0; \vec{F}_g \perp d\vec{r} \\ W_n = 0; \vec{F}_n \perp d\vec{r} \end{array} \right.$$

$$\int_{-X_i}^{X_f} \vec{F}_{S_1} \cdot d\vec{r} + \int_0^{X_f} \vec{F}_{S_2} \cdot d\vec{r} = 0$$

$$\int_{-X_i}^{X_f} -k_1 x \hat{i} \cdot dx \hat{i} + \int_0^{X_f} -k_2 x \hat{i} \cdot dx \hat{i} = 0$$

$$\left\{ \begin{array}{l} \vec{F}_s = -kx \hat{i} \\ d\vec{r} = dx \hat{i} + dy \hat{j} \end{array} \right.$$

$\left\{ \begin{array}{l} k_1 + k_2 \\ \text{are constant} \end{array} \right.$

$$-k_1 \int_{-X_i}^{X_f} x dx + -k_2 \int_0^{X_f} x dx = 0$$

$$-\frac{1}{2} k_1 x^2 \Big|_{-X_i}^{X_f} + -\frac{1}{2} k_2 x^2 \Big|_0^{X_f} = 0$$

$$-\frac{1}{2} k_1 (X_f^2 - (-X_i)^2) + -\frac{1}{2} k_2 (X_f^2 - 0) = 0$$

$$k_1 X_f^2 - k_1 X_i^2 + k_2 X_f^2 = 0$$

$\left\{ \begin{array}{l} \text{solve for } X_f \end{array} \right.$

$$(k_1 + k_2) X_f^2 = k_1 X_i^2$$

$$X_f^2 = \frac{k_1}{k_1 + k_2} X_i^2$$

$$X_f = \sqrt{\frac{k_1 X_i^2}{k_1 + k_2}}$$