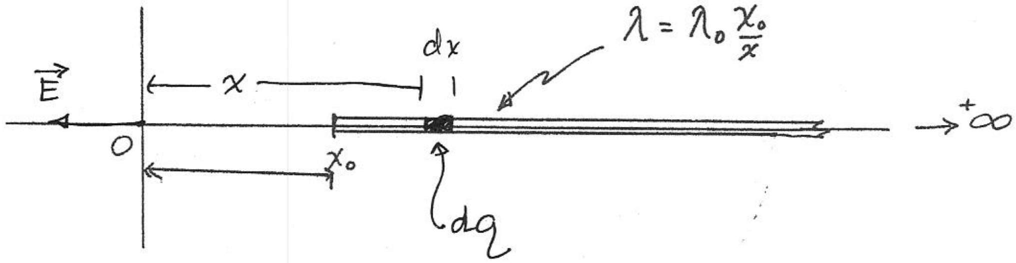
Physics 4B Winter 2015 - Dickson Name *Solutions*

Exam 1

Show **all** your work for full credit. Scratch paper, note cards, calculators, or electronic devices are not allowed.

1. A line charge starts at $x = +x_o$ and extents to positive infinity. The linear charge density is $\lambda = \lambda_o x_o/x$, where λ_o is a constant. Determine the electric field at the origin.



$$dq = \lambda\, dx$$

$$\vec{E} = \int d\vec{E}$$

$$\left[\vec{E} = \int \frac{k\, dq}{x^2}(-\hat{\imath})\right] \cdot \hat{\imath}$$

$$E_x = -\int_{x_0}^{\infty} k \frac{\lambda\, dx}{x^2}$$

$$E_x = -\int_{x_0}^{\infty} k \frac{\lambda_0 x_0}{x^3}\, dx$$

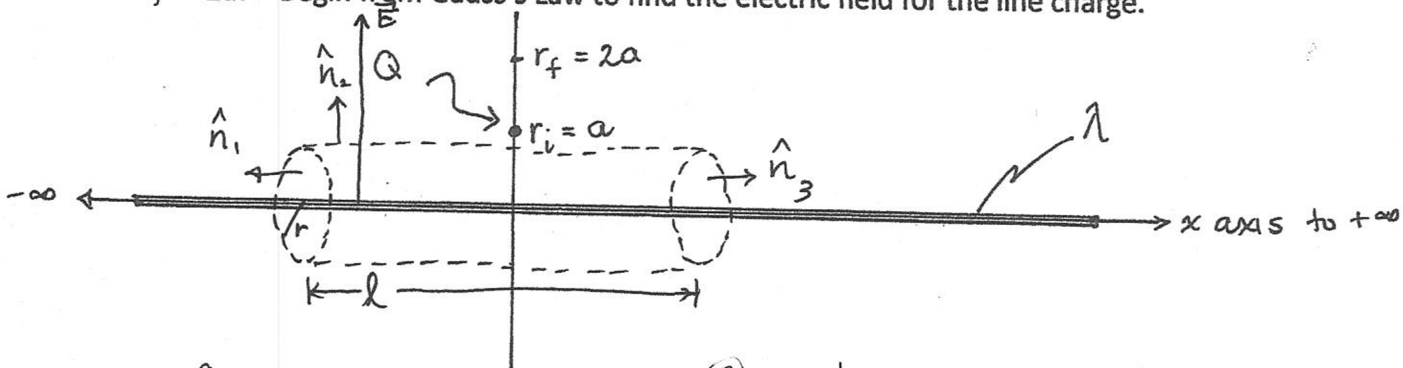
$$E_x = -k\lambda_0 x_0 \frac{-1}{2} x^{-2} \Big|_{x_0}^{\infty}$$

$$E_x = \frac{k\lambda_0 x_0}{2}\left[\cancelto{0}{\frac{1}{\infty^2}} - \frac{1}{x_0^2}\right]$$

$$E_x = -\frac{k\lambda}{2x_0}$$

$$\boxed{\vec{E} = -\frac{k\lambda}{2x_0}\hat{\imath}}$$

2. An infinitely long line of charge, with charge per unit length equal to λ, lies along the *x-axis*. A point charge Q, of mass m, is initially at rest and located distance a above the line of charge. The point charge Q is then released. What is its speed when the distance from the line of charge is doubled? That is, when it has moved from $r_i = +a$ to $r_f = +2a$. Begin from Gauss's Law to find the electric field for the line charge.



① $\Phi_e = \frac{q_{in}}{\epsilon_0}$

$\oint \vec{E}\cdot\hat{n}\,dA = \frac{\lambda \ell}{\epsilon_0}$

$\int_1 \vec{E}\cdot\cancel{\hat{n}_1}^{\,0}dA_1 + \int_2 \vec{E}\cdot\hat{n}_2 dA_2 + \int_3 \vec{E}\cdot\cancel{\hat{n}_3}^{\,0}dA_3 = \frac{\lambda \ell}{\epsilon_0}$

$\vec{E}\perp\hat{n}_1 \qquad \vec{E}\parallel\hat{n}_2 \qquad \vec{E}\perp\hat{n}_3$

$\int E_r\, dA_2 = \frac{\lambda \ell}{\epsilon_0}$

$E_r\, 2\pi r \cancel{\ell} = \frac{\lambda \cancel{\ell}}{\epsilon_0}$

$E_r = \frac{\lambda}{2\pi\epsilon_0 r}$

$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$

② $\Delta V = -\int \vec{E}\cdot d\vec{s}$

$V_f - V_i = -\int_{r_i}^{r_f} \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}\cdot d\vec{s}$

$V_f - V_i = -\int_{r_i}^{r_f} \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}\cdot dr\,\hat{r}$

$V_f - V_i = -\int_{r_i}^{r_f} \frac{\lambda}{2\pi\epsilon_0}\frac{dr}{r}$

$V_f - V_i = -\frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_f}{r_i}$

② cont.

$r_f = 2a\ ;\ r_i = a$

$V_f - V_i = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{2a}{a}\right)$

$V_f - V_i = -\frac{\lambda}{2\pi\epsilon_0}\ln 2$

③ system: Q and line charge

$W_{net} = \Delta E$

$0 = \Delta U_e + \Delta K$

$0 = q\Delta V + K_f - \cancel{K_i}^{\,0}$

$K_f = -Q\Delta V$

$\frac{1}{2}mv_f^2 = -Q\left(-\frac{\lambda}{2\pi\epsilon_0}\ln 2\right)$

$mv_f^2 = \frac{Q\lambda}{\pi\epsilon_0}\ln 2$

$\boxed{v_f = \sqrt{\frac{Q\lambda \ln 2}{m\pi\epsilon_0}}}$

③ alt. w/ gravity syst. Q, line charge, earth

$W_{net} = \Delta E$

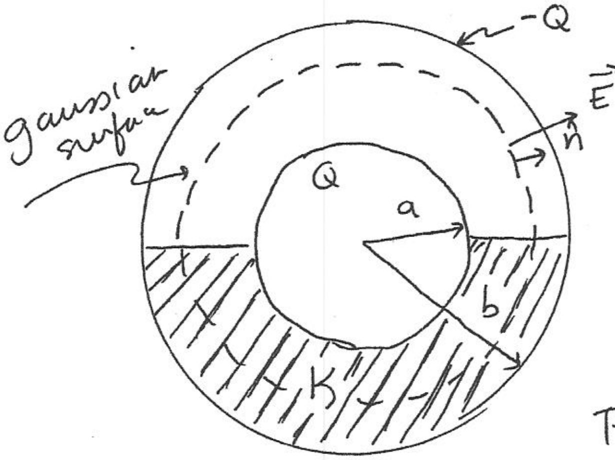
$0 = \Delta U_e + \Delta U_g + \Delta K$

$0 = q\Delta V + U_{gf} - U_{gi} + K_f - \cancel{K_i}^{\,0}$

$K_f = -Q\Delta V + mga - mg2a$

$v_f = \sqrt{\frac{Q\lambda \ln 2}{m\pi\epsilon_0}} - ga$

3. A spherical capacitor consists of a spherical conducting shell of radius b and charge $-Q$ concentric with a smaller conducting sphere of radius a and charge Q. The space between the two spheres is half-filled with a dielectric material of strength κ as shown. Find the capacitance of this device.



Since the inner and outer spheres are conductors, they are each equipotential surfaces

$$\Delta V = -\int_a^b \vec{E}_{Top} \cdot d\vec{\ell}$$

$$\Delta V = -\int_a^b \vec{E}_{bottom} \cdot d\vec{\ell}$$

Thus $E_{bottom} = E_{Top}$

Using Gauss's Law, we find $\vec{E}$ between the spheres as if no dielectric is present.

$$\oint \vec{E} \cdot \hat{n}\, dA = \frac{q_{in}}{\epsilon_0} \qquad \vec{E} \parallel \hat{n}$$

$$E_r \oint dA = \frac{Q}{\epsilon_0}$$

$$E\, 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E_r = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\Delta V = -\int_a^b \vec{E} \cdot d\vec{s}$$

$$\Delta V = -\int_a^b \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \cdot dr\, \hat{r}$$

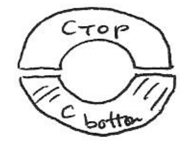
$$\Delta V = -\int_a^b \frac{Q\, dr}{4\pi\epsilon_0 r^2}$$

$$\Delta V = +\frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_a^b$$

$$\Delta V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$\frac{1}{b} - \frac{1}{a} < 0 \qquad |\Delta V| = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{4\pi\epsilon_0 ab}{b-a}$$

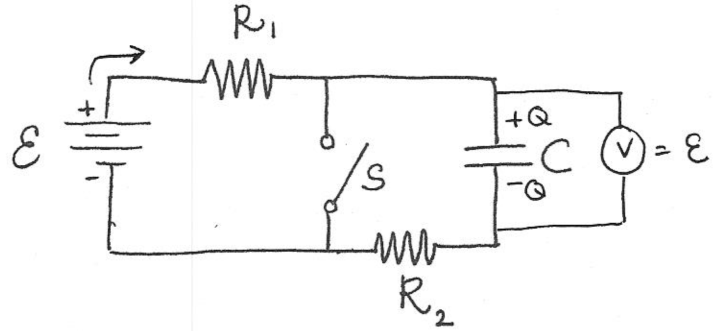


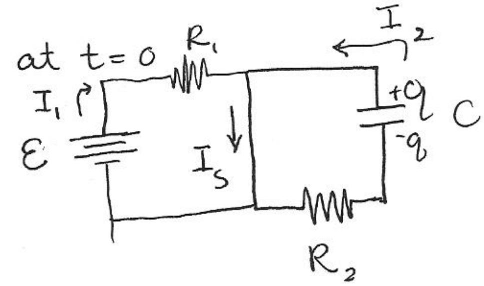
$$C_{eq} = C_{Top} + C_{BOTTOM}$$

$$C_{Top} = \frac{1}{2}C \; ; \; C_{bottom} = \frac{1}{2}KC$$

$$\boxed{C_{eq} = \frac{2\pi\epsilon_0 ab}{b-a}\left[1 + K\right]}$$

4. In the circuit shown, the switch has been open for a long time. It is then suddenly closed at t = 0. Determine the current in the switch as a function of time.





(1) $\mathcal{E} - I_1 R_1 = 0$

(2) $\dfrac{q}{C} - I_2 R_2 = 0$

(3) $I_1 + I_2 = I_S$

FROM (1) $I_1 = \dfrac{\mathcal{E}}{R_1}$

FROM (2) $\dfrac{q}{C} = I_2 R_2$

but $I_2 = -\dfrac{dq}{dt}$

$$\frac{q}{C} = -\frac{dq}{dt} R_2$$

$$\int_{Q_{max}}^{q} \frac{dq}{q} = \int_0^t -\frac{dt}{R_2 C}$$

$$\ln \frac{q}{q_{max}} = \frac{-t}{R_2 C}$$

$$\frac{q}{q_{max}} = +e^{-t/RC}$$

$$q = +q_{max} e^{-t/RC}$$

$$I_2 = -\frac{dq}{dt} = -\left(-\frac{q_{max}}{R_2 C} e^{-t/R_2 C}\right)$$

$$I_2 = \frac{\mathcal{E}C}{R_2 C} e^{-t/R_2 C}$$

$$I_2 = \frac{\mathcal{E}}{R_2} e^{-t/R_2 C}$$

From (3)

$$\boxed{I_S = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} e^{-t/R_2 C}}$$