Physics 4B Winter 2016 Exam 1

Name Solutions

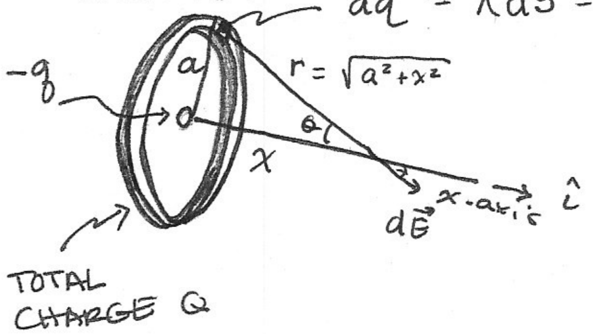
Dickson

Show all your work for full credit. No note cards, scratch papers or electronic devices are allowed. You have one (1) hour to complete all four (4) questions.

1. (25 points). A negatively charged particle -*q* is placed at the center of a uniformly charged ring which has positive charge *Q* as shown. (Let the uniform charge density per unit length be λ.) The particle, of mass m, confined to move along the *x* axis, is moved a small distance *x* along the axis (where x << a *{this is a hintg}*) and released. Show that the particle oscillates in simple harmonic motion and find the frequency (f) of oscillation. (Recall, ω = 2πf) (For full credit, you must begin by deriving an expression for the force on the charge.)

$$dq = \lambda ds = \lambda a\, d\phi \qquad \lambda = \frac{Q}{L} = \frac{Q}{2\pi a}$$

$$\cos\theta = \frac{x}{r}$$



System : $-q$

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F}_e = m\vec{a}$$

$$\hat{i}: \quad -qE = ma$$

$$-\frac{kqQx}{a^3} = m\frac{d^2x}{dt^2}$$

let $x = A\sin\omega t$

$$\frac{dx}{dt} = A\omega\cos\omega t$$

$$\frac{d^2x}{dt^2} = -A\omega^2\sin\omega t$$

$$+\frac{kqQ}{ma^3}(A\sin\omega t) = +A\omega^2\sin\omega t$$

$$\omega = \sqrt{\frac{kQq}{ma^3}}$$

$$\boxed{f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{kQq}{ma^3}}}$$

$$\vec{E} = \int d\vec{E}$$

$$\vec{E} = \int (dE_x\hat{i} + dE_{\perp}\hat{j})$$

($dE_{\perp}$ term → 0 by symmetry)

$$E_x = \int \frac{k\,dq}{r^2}\cos\theta$$

$$E_x = \int_0^{2\pi} \frac{k\lambda a\, d\phi \cos\theta}{r^2}$$

$$E_x = \frac{k\lambda a x}{r^3}\phi\Big|_0^{2\pi}$$

$$\vec{E} = \frac{k 2\pi\lambda a x}{r^3}\hat{i}$$

$$\vec{E} = \frac{kQx}{(x^2+a^2)^{3/2}}\hat{i}$$

for $x \ll a$

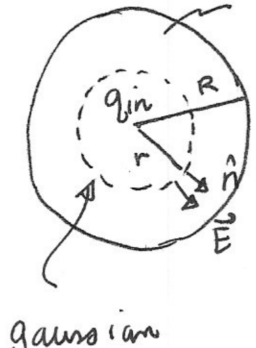
$$\vec{E} \cong \frac{kQx}{a^3}\hat{i}$$

2. (A non-uniform, but spherically symmetric, distribution of charge has a charge density ρ (r) given as: $\rho = \rho_o(1 - r/R)$ for $r \le R$

$P = 0$ for $r \ge R$

Starting from Gauss's Law, find an expression for the electric field in the region $r \le R$. Use this expression to find the value of *r* at which the electric field is a maximum. Your answer should be in terms of *R*.

charge density ρ



$$\oint \vec{E} \cdot \hat{n}\, dA = \frac{q_{in}}{\epsilon_0}$$

$$\oint E_r \hat{r} \cdot \hat{r}\, dA = \int\!\!\int \frac{\rho\, dv}{\epsilon_0}$$

$$E_r\, 4\pi r^2 = \int_0^r \frac{\rho_o}{\epsilon_0} \left(1 - \frac{r}{R}\right) 4\pi r^2 dr$$

$$E_r = \frac{1}{4\pi r^2} \frac{\rho_o}{\epsilon_0} \cdot 4\pi \left[\int_0^r r^2 dr - \int_0^r \frac{r^3}{R} dr \right]$$

$$E_r = \frac{\rho_o}{\epsilon_0 r^2} \left[\frac{r^3}{3}\Big|_0^r - \frac{r^4}{4R}\Big|_0^r \right]$$

$$E_r = \frac{\rho_o}{\epsilon_0} \left(\frac{r^3}{3r^2} - \frac{r^4}{4r^2 R} \right)$$

$$E_r = \frac{\rho_o}{\epsilon_0} \left(\frac{r}{3} - \frac{r^2}{4R} \right)$$

$$\frac{dE_r}{dr} = 0 = \frac{d}{dr} \left(\frac{\rho_o}{\epsilon_0} \left(\frac{r}{3} - \frac{r^2}{4R} \right) \right)$$

$$\frac{1}{3} - \frac{r}{2R} = 0$$

$$\boxed{r = \frac{2R}{3}}$$

This method is acceptable

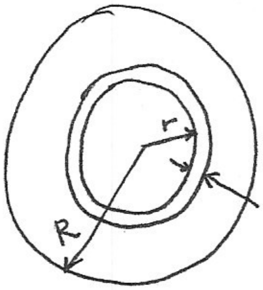
This way is difficult, but interesting and a better estimate of the work ↙

3. (25 points) Find the total work required to charge one surface of a circular dielectric disk of radius R uniformly with a total charge of Q. (*Hint:* Let the uniform surface charge density be σ. Find the work required to bring the charge dq from infinity to the circular ring of radius r and thickness dr against the charge already present on the disk of radius r. Then integrate from zero to R to find the total potential energy.) Your answer should be in terms of K, Q and R

$dq = \sigma 2\pi r\,dr$

$q = \sigma \pi r^2$

$Q = \sigma \pi R^2$



Method 1: approximation

Let the charge on the disk be very, very small ≈ point charge

$W = \int V\,dq$

$W \cong \int \frac{kq}{r}\,dq$

$W \cong \int \frac{k(\sigma\pi r^2)(\sigma 2\pi r\,dr)}{r}$

$W \cong 2k\sigma^2\pi^2 \int_0^R r^2\,dr$

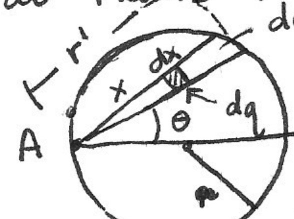
$W \cong 2k\sigma^2\pi^2 \frac{r^3}{3}\Big|_0^R$

$W \cong \frac{2k\sigma^2\pi^2 R^3}{3}$ $\left(\sigma^2 = \frac{Q^2}{\pi^2 R^4}\right)$

$\boxed{W \cong \frac{2kQ^2}{3R}}$

either way!

Method 2: Find V for a disk at radius r at the point A.



$\sigma = \frac{dq}{dA} = \frac{dq}{x\,dx\,d\theta}$

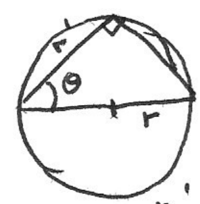
let the radius be r

$dV = \int \frac{k\,dq}{x}$

$dV = \int_0^{r'} \frac{k\sigma\,dx\,x\,d\theta}{x}$

$dV = k\sigma\,d\theta\,x\Big|_0^{r'}$

$dV = k\sigma\,d\theta\,r'$



$V = \int_{-\pi/2}^{\pi/2} k\sigma\,2r\cos\theta\,d\theta$

$\cos\theta = \frac{r'}{2r}$

$r' = 2r\cos\theta$

$V = k\sigma 2r\left[\sin\theta\Big|_{-\pi/2}^{\pi/2}\right]$

$V = k\sigma 2r(+1+1)$

$V = 4k\sigma r$

$W = \int V\,dq$

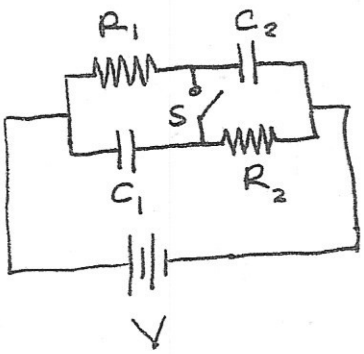
$W = \int_0^R 4k\sigma r \cdot \sigma 2\pi r\,dr$

$W = 8k\pi\sigma^2 \int_0^R r^2\,dr$

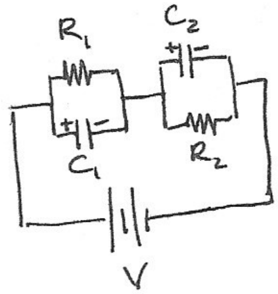
$\boxed{W = 8k\pi\sigma^2 \frac{r^3}{3}\Big|_0^R = \frac{8k\pi\sigma^2 R^3}{3} = \frac{8kQ^2}{3\pi R}}$

4. (25 points) Switch S shown in the figure has been closed a long time and the electric circuit carries a constant current. The voltage of the source is V, and the resistors and capacitors have the values C_1, C_2, R_1 and R_2. The switch is now opened. After a long time with the switch open ($t \to \infty$) by what fraction has the charge Q_2 on capacitor C_2 changed? (Please tell me the ratio of Q_{2f}/Q_{2i})

$C = \frac{Q}{V}$



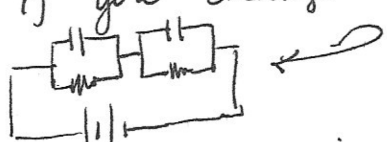
initially S closed for a long time



$V = IR_1 + IR_2$

$V = I(R_1 + R_2)$

Initially, the capacitors are connected in series. A bit easier to see if you change the drawing to



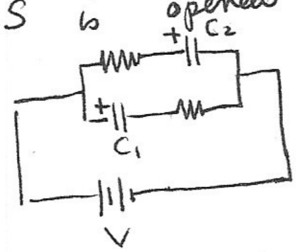
Notice the $C_2(+)$ plate is connected to the $C_1(-)$ plate

Therefore

$Q_i = Q_{1i} = Q_{2i}$ since $V_1 + V_2 = V$

$\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_{series}}$ $\qquad \frac{Q_{1i}}{C_1} + \frac{Q_{2i}}{C_2} = \frac{Q_i}{C_{series}}$

Then S is opened for $t \to \infty$



The open switch for a long time implies $I \to 0$ Now the caps are in parallel. The + plates are both on the same side. Here the potential drops across each cap are the same

$V = \frac{Q_{1f}}{C_1} = \frac{Q_{2f}}{C_2}$

$\frac{Q_i}{C_S} = V = \frac{Q_{2f}}{C_2}$ $\qquad \frac{Q_{2f}}{Q_{2i}} = \frac{C_2}{C_S}$

$\frac{1}{C_S} = \frac{C_2 + C_1}{C_1 C_2}$

$C_S = \frac{C_1 C_2}{C_1 + C_2}$

$\boxed{\frac{Q_{2f}}{Q_{2i}} = \frac{C_2(C_1 + C_2)}{C_1 C_2} = \frac{C_1 + C_2}{C_1}}$