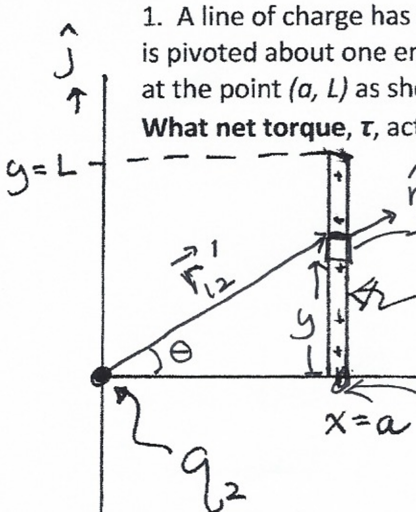
Physics 4B Winter 2019 Name SOLUTION

Exam 1

Please show all your work for full credit. No scratch papers, calculators, cell phones, or other electronic devices are allowed. You have one (1) hour for four (4) problems.

1. A line of charge has uniform positive charge per unit length λ, total charge q_1, and length L. This line is pivoted about one end at point $x = a$ from the origin. The opposite end is free to move and is initially at the point (a, L) as shown in the diagram. A positive point charge, q_2, is held in place at the origin. **What net torque, τ,** acts on the line of charge? You may ignore gravity.



$dq_1 = \lambda dy$ system: line charge

$$\lambda = \frac{dq_1}{dy} = \frac{q_1}{L}$$

$$\vec{\tau}_{net} = \int d\vec{\tau} \qquad \hat{r}_{12} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\vec{\tau}_{net} = \int \vec{r} \times d\vec{F}$$

this $\rightarrow$ $\frac{kq_2 dq_2}{r_{12}^2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & y & 0 \\ \cos\theta & \sin\theta & 0 \end{vmatrix}$

$$\vec{\tau}_{net} = \int y\hat{j} \times \frac{kq_2\, dq_1}{r_{12}^2}\hat{r}_{12}$$

or this $y\hat{j}$ $\xrightarrow{90-\theta}$ $\hat{r}_{12}$

$$\vec{\tau}_{net} = \int \frac{y\, kq_2\, dq_1}{r_{12}^2}(-\cos\theta)\hat{k}$$

$|y\hat{j}||\hat{r}_{12}|\sin(90-\theta)$, dir. by r.h.r.

$$\vec{\tau}_{net} = -kq_2\hat{k}\int \frac{y\,\lambda dy\, a}{(a^2+y^2)^{3/2}}$$

let $u = y^2 + a^2$

$du = 2y\,dy$

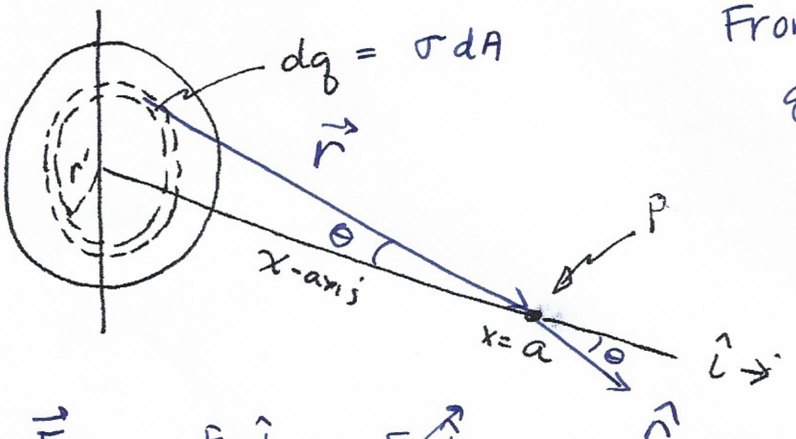
$$\frac{1}{2}\int \frac{du}{u^{3/2}} = \frac{\frac{1}{2}u^{-1/2}}{-1/2}\Big|$$

$$\vec{\tau}_{net} = -kq_2\lambda a\hat{k}\left[\frac{-1}{\sqrt{a^2+y^2}}\right]_0^L$$

$$\vec{\tau}_{net} = -kq_2\lambda a\hat{k}\left[\frac{1}{\sqrt{a^2}} - \frac{1}{\sqrt{a^2+L^2}}\right]$$

$$\boxed{\vec{\tau}_{net} = \frac{kq_1q_2}{L}\left[1 - \frac{a}{\sqrt{a^2+L^2}}\right](-\hat{k})}$$

2. A disk of radius R carries a surface charge density $\sigma = \sigma_o\,(r'/R)$. **Find the electric field vector *E*** on the axis of the disk a distance $x = a$ from the center of the disk. For full credit, you may leave your answer as an integral ready to integrate.



From finding the Electric field for the ring of charge, we showed $E_y \to 0$ by symmetry

$$\vec{E} = E_x\,\hat{\imath} + \cancel{E_y\,\hat{\jmath}}$$

$$E_x = \int dE_x$$

$$E_x = \int \frac{k\,dq}{r^2}\cos\theta$$

$$E_x = \int \frac{k\,\sigma\, dA\, a}{\sqrt{(a^2 + r'^2)}^{3/2}}$$

$$E_x = \int_0^R \frac{k\sigma_o r'}{R}\cdot\frac{2\pi r'\,dr'\,a}{(a^2 + r'^2)^{3/2}}$$

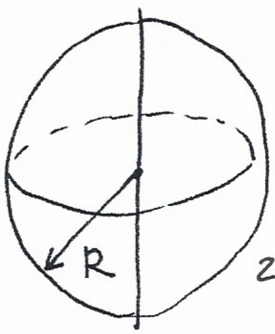
$$\boxed{\vec{E} = \frac{2\pi k\, a\,\sigma_o}{R}\int_0^R \frac{r'^2\,dr'}{(a^2 + r'^2)^{3/2}}\;\hat{\imath}}$$

3. A dielectric sphere of radius R contains a total charge Q of uniform volume charge density ρ. The electric field inside the sphere is given by $\boldsymbol{E} = \frac{kQr}{R^3}\hat{\boldsymbol{r}}$

Find the electric potential V, as a function of r for r < R.

$$\vec{E}_{inside} = \frac{kQr}{R^3}\hat{r}$$

$$\vec{E}_{outside} = \frac{kQ}{r^2}\hat{r}$$



V inside = ?

$$\Delta V = -\int \vec{E}\cdot d\vec{\ell}$$

$$\cancel{V(\infty)}^{0} - V(r) = -\int_r^{\infty} \vec{E}\cdot d\vec{\ell}$$

2 minus signs

$$+V(r) = +\left[\int_r^R \vec{E}_{in}\cdot d\vec{\ell} + \int_R^{\infty} \vec{E}_{out}\cdot d\vec{\ell}\right]$$

$$V(r) = \left[\int_r^R \frac{kQr}{R^3}\hat{r}\cdot dr\,\hat{r} + \int_R^{\infty} \frac{kQ}{r^2}\hat{r}\cdot dr\,\hat{r}\right]$$

$$V(r) = \left[\frac{kQ}{R^3}\int_r^R r\,dr + kQ\int_R^{\infty}\frac{dr}{r^2}\right]$$

$$V(r) = \left[\frac{kQ}{R^3}\left(\frac{r^2}{2}\Big|_r^R\right) + kQ\left(\frac{-1}{r}\Big|_R^{\infty}\right)\right]$$

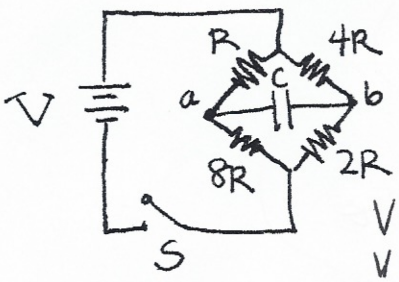
$$V(r) = \left(\frac{kQ}{R^3}\left(\frac{R^2}{2} - \frac{r^2}{2}\right) + \frac{kQ}{R}\right)$$ ← as expected!

$$V(r) = kQ\left(\frac{1}{2R} + \frac{1}{R} - \frac{r^2}{2R^3}\right)$$

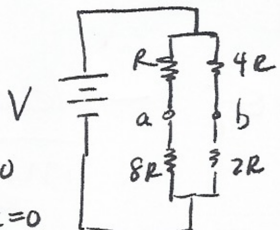
$$\boxed{V(r) = \frac{kQ}{2R^3}\left(3R^2 - r^2\right)}$$

4. In the circuit shown switch *S* has been closed for a long time and the capacitor, *C*, is fully charged.

a. **What is the electric potential V_{ab}** across the capacitor?



if C is charged



$V - I_1 R - I_1 8R = 0$

$V - I_2 4R - I_2 2R = 0$

$I_1 = \frac{V}{9R}$; $I_2 = \frac{V}{6R}$

$V_a = V - I_1 R = V - \frac{V}{9\cancel{R}}\cdot\cancel{R}$

$V_a = \frac{8}{9}V$

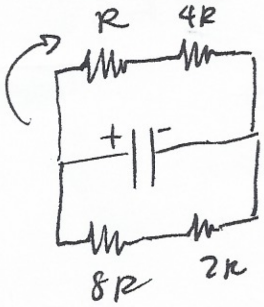
$V_b = V - I_2 4R = V - \frac{V}{3\cancel{6R}}\cdot {}^{2}\cancel{4R}$

$V_b = \frac{1}{3}V$

$V_{ba} = V_a - V_b = \frac{8}{9}V - \frac{1}{3}V = \frac{5}{9}V$

$V_{ab} = -\frac{5}{9}V$

b. At time $t = 0$, switch *S* is opened. **Find the charge on the capacitor as a function of time.**



$\frac{Q}{C} - I_{Top} R - I_{Top} 4R = 0$

$\frac{Q}{C} - I_{BOT} 8R - I_{BOT} 2R = 0$

$\frac{Q}{C} - I_{TOP} 5R = 0$

$\frac{Q}{C} - I_{BOT} 10R = 0$

$I_{Top}\,\cancel{5R} = I_{BOT}\,{}^{2}\cancel{10R}$

$I_{Top} = 2I_{BOT}$

$I_{cap} = I_{Top} + I_{BOT}$

$I_{CAP} = 2I_{BOT} + I_{BOT}$

$I_{CAP} = 3I_{BOT}.$

$\frac{Q}{C} = \frac{I_{CAP}}{3}\cdot 10R$

$\frac{3Q}{10RC} = I_{cap}$

$\frac{3Q}{10RC} = -\frac{dQ}{dt}$

$\int_{Q_{max}}^{Q}\frac{dQ}{3Q} = \int_{0}^{t}-\frac{dt}{10RC}$

$\frac{1}{3}\ln\frac{3Q}{3Q_{max}} = \frac{-t}{10RC}$

$\frac{\cancel{3}Q}{\cancel{3}Q_{max}} = e^{-3t/10RC}$

$\boxed{Q = Q_{max}e^{-3t/10RC}}$

$Q = \frac{5}{9}VCe^{-3t/10RC}$