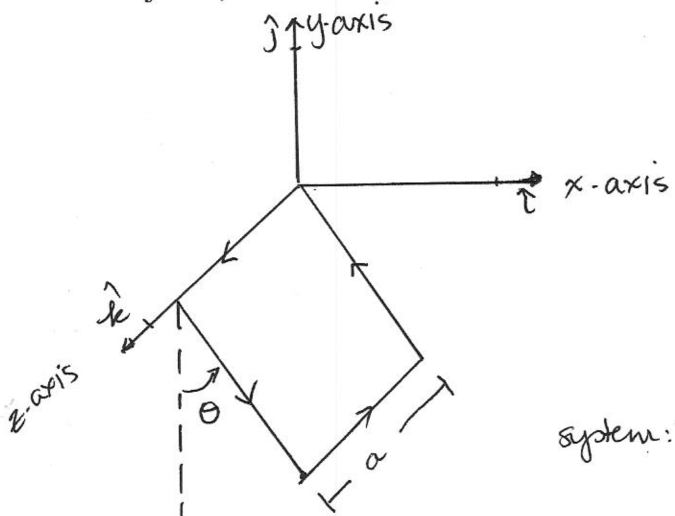
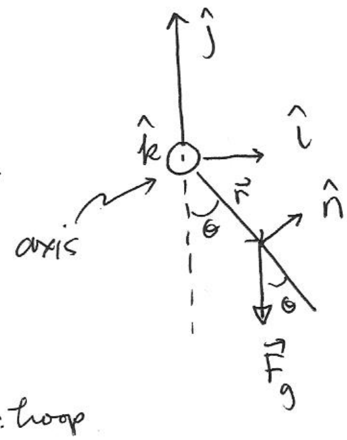
Physics 4B Winter 2015 - EXAM 2
Dickson

Name Solutions

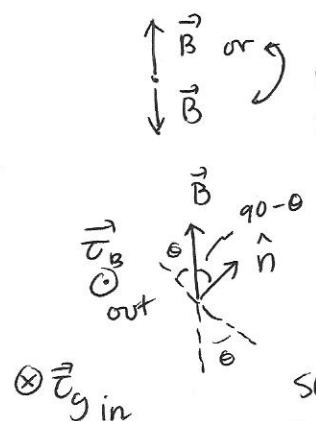
Show all your work for full credit. No calculators, note cards, scratch paper or electronic devices are allowed during the exam. Complete all four questions. You have one hour.

1. The single, square loop of wire shown in the figure has a total mass of M and sides of length a. It is hinged about a frictionless axis along the z-axis at the top of the loop. The current in the wire is I. The magnetic field in the region is uniform and is parallel to the y-axis. If the loop makes an angle θ with the vertical, what is the magnitude and direction (positive or negative j-hat?) of the magnetic field, B? (DO NOT IGNORE GRAVITY IN THIS PROBLEM)





system: loop



which way?

must be $+\hat{j}$ so that the torques balance.

$$\vec{\tau}_{net} = I\vec{\alpha}$$

$$\vec{\tau}_g + \vec{\tau}_B = I\cancel{\vec{\alpha}}^{\,0}$$

$$\vec{r} \times \vec{F}_g + \vec{\mu} \times \vec{B} = 0$$

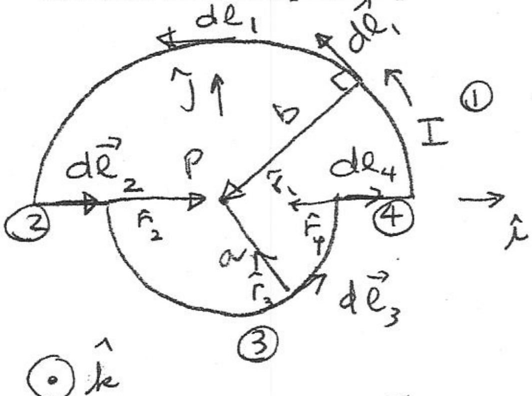
out (+): $$-\frac{a}{2} Mg \sin\theta + NIA\left|\left(\hat{n} \times B\hat{j}\right)\right| = 0 \qquad (N=1)$$

$$\frac{\cancel{a}}{2} Mg \sin\theta = Ia^{\cancel{2}} B \sin(90-\theta)$$

$$B = \frac{Mg \sin\theta}{2Ia\cos\theta}$$

$$\boxed{\vec{B} = \frac{Mg \tan\theta}{2Ia}\,\hat{j}}$$

2. A closed circuit with radii a and b as shown carries current I. What is the magnitude and direction of the magnetic field B at point P. (For full credit, you must begin with the Biot-Savart Law and show complete steps for all four segments of the wire.)



$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi}\frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\left[\int_1 \frac{I\, d\vec{\ell}_1 \times \hat{r}_1}{b^2} + \int_2 \cancel{\frac{I\, d\vec{\ell}_2 \times \hat{r}_2}{b^2}}^{\,0} + \int_3 \frac{I\, d\vec{\ell}_3 \times \hat{r}_3}{a^2} + \int_4 \cancel{\frac{I\, d\vec{\ell}_4 \times \hat{r}_4}{b^2}}^{\,0}\right]$$

$d\vec{\ell}_1 \perp \hat{r}_1$ $\quad$ $d\vec{\ell}_2 \parallel \hat{r}_2$ $\quad$ $d\vec{\ell}_3 \perp \hat{r}_3$ $\quad$ $d\vec{\ell}_4 \parallel \hat{r}_4$

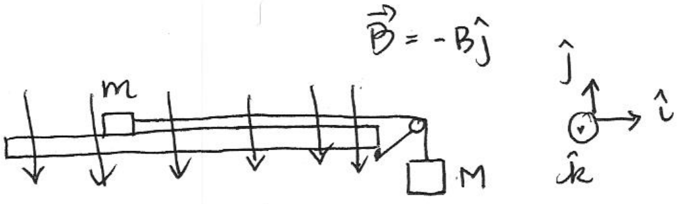
$d\vec{\ell}_1 \times \hat{r}_1 \to$ out $(\hat{k})$ $\quad$ $d\vec{\ell}_3 \times \hat{r}_3 \to$ out $(\hat{k})$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\left[\int \frac{d\ell_1\, \hat{k}}{b^2} + \int \frac{d\ell_3\, \hat{k}}{a^2}\right]$$

$$B_z = \frac{\mu_0 I}{4\pi}\left[\int_0^{\pi} \frac{\cancel{b}\, d\theta}{b^{\cancel{2}}} + \int_{\pi}^{2\pi} \frac{\cancel{a}\, d\theta}{a^{\cancel{2}}}\right]$$

$$B_z = \frac{\mu_0 I}{4\pi}\left[\frac{\pi}{b} + \frac{2\pi - \pi}{a}\right]$$

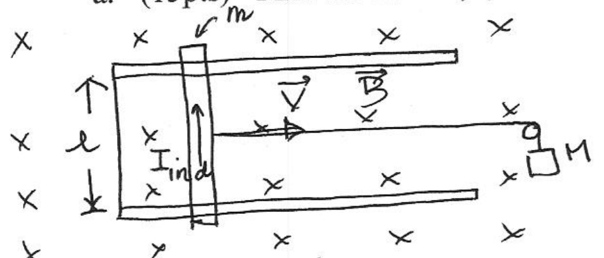
$$\boxed{\vec{B} = \frac{\mu_0 I}{4}\left[\frac{1}{b} + \frac{1}{a}\right]\hat{k}}$$



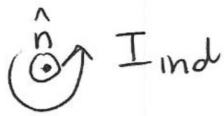
SIDE VIEW

3. A conducting bar of mass *m*, length *l* and resistance *R*, is connected to a hanging mass *M* by a massless, ideal string. The bar is on frictionless conducting rails which form a closed loop and the loop is itself immersed in a vertical magnetic field ***B*** = ***-Bj-hat*** as shown. The system is released from rest.

a. (15pts) Find the current, *I*, induced in the bar.



TOP VIEW



$$I_{ind} = \frac{\mathcal{E}_{ind}}{R}$$

$$I_{ind} = \frac{1}{R}\left(-\frac{d\Phi_B}{dt}\right)$$

$$I_{ind} = -\frac{1}{R}\left(\frac{d}{dt}\int \vec{B}\cdot\hat{n}\,da\right)$$

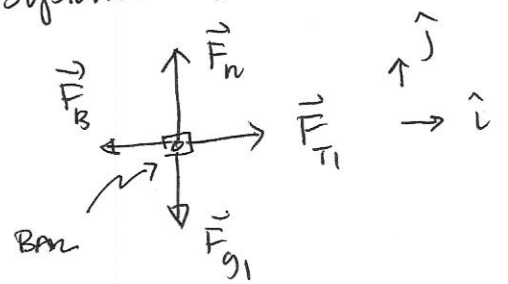
$$I_{ind} = \frac{-1}{R}\frac{d}{dt}\int -B\hat{j}\cdot\hat{j}\,da$$

$$I_{ind} = \frac{1}{R}Bl\frac{d}{dt}\int dx$$

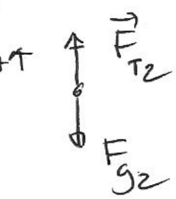
$$\boxed{I_{ind} = \frac{Blv}{R}}$$

b. (10pts) Find the maximum velocity attained by the bar (the terminal velocity).

System: bar



System: hanging mass



$$\vec{F}_{net_2} = m_2\vec{a}^{\,0}$$

$$\vec{F}_{T2} + \vec{F}_{g2} = 0$$

$$+: F_{T2} = Mg$$

$$\vec{F}_{net_1} = m_1\vec{a}$$

$$\vec{F}_B + \vec{F}_n + \vec{F}_{T1} + \vec{F}_{g1} = 0$$

$$\left(I_{ind}\vec{l}\times\vec{B}\right) + \vec{F}_n + \vec{F}_{T1} + \vec{F}_{g1} = 0$$

$$\hat{i}: -I_{ind}lB\sin 90^\circ + F_{T1} = 0 \qquad |\vec{F}_{T1}| = |\vec{F}_{T2}| = F_T$$

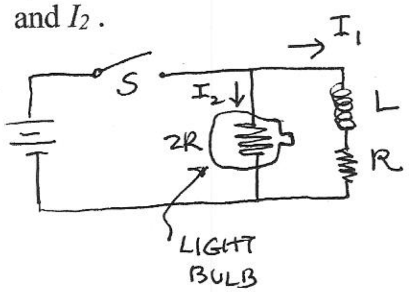
$$I_{ind}lB = Mg$$

$$\frac{B^2l^2v_{max}}{R} = Mg$$

$$\boxed{v_{max} = \frac{RMg}{B^2l^2}}$$

4. In the circuit shown, the switch is initially closed and the light bulb just barely glows. After a long time, the switch is opened and the bulb lights up briefly for a short period of time.

a) Switch S has been closed for a long time. I_1 and I_2 have reached steady state values. Find I_1 and I_2.



$V_L \to 0$

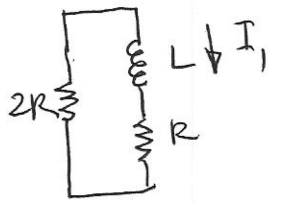
$\mathcal{E} - I_1 R = 0$

$\mathcal{E} - I_2 2R = 0$

$$\boxed{I_1 = \frac{\mathcal{E}}{R}}$$

$$\boxed{I_2 = \frac{\mathcal{E}}{2R}}$$

b) Switch S is now opened, Obtain (derive - starting from Kirchoff's Loop Theorem) an expression for the current through the inductor and the light bulb as an explicit function of time. (Recall, $V_L = -LdI/dt$. Hint: All terms will have the same sign resulting in an exponential decay function.)



$$V_L - IR - I2R = 0$$

$$-L\frac{dI}{dt} - I3R = 0$$

$$\frac{dI}{dt} = -\frac{3R}{L} I$$

$$\int_{I_1}^{I} \frac{dI}{I} = \int_0^t -\frac{3R}{L}\, dt$$

$$\ln\frac{I}{I_1} = -\frac{3R}{L} t$$

$$\frac{I}{I_1} = e^{-\frac{3Rt}{L}}$$

$$I = I_1 e^{-3Rt/L}$$

$$\boxed{I = \frac{\mathcal{E}}{R} e^{-3Rt/L}}$$