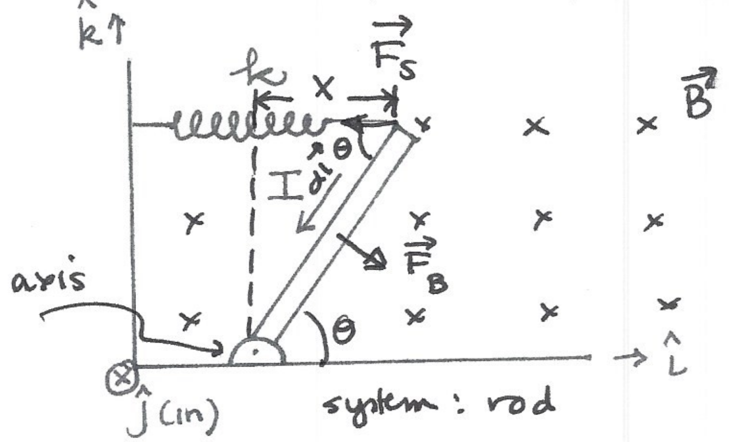
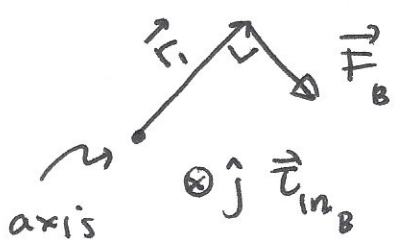
4B Exam 2 – Winter 2017 Name SOLUTION

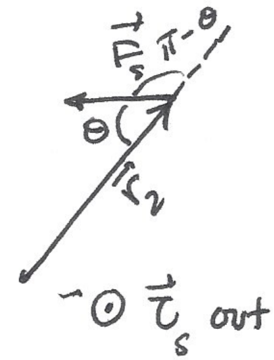
Dickson

Show all your work for full credit. No scratch papers, note cards, cell phones, calculators or electronic devices are allowed. You have one (1) hour.

1. A thin, uniform rod of length L rests on a horizontal frictionless surface. It is hinged along one end and the other end is attached to a spring of force constant k. The rod carries current I as shown. Initially, there is no magnetic field and the spring is unstretched. When a uniform, vertical magnetic field $\boldsymbol{B}$ is established in the region, the rod stretches the spring and the angle the rod makes with the pivot point is θ. Find the magnitude and direction (*+j-hat* or *–j-hat*) of magnetic field. (Note: since the surface is horizontal, there is not torque due to gravity in the problem.)







+

$$\vec{\tau}_{net} = I\vec{\alpha}$$

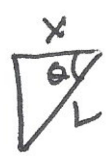
$$\vec{F}_B = \int I\,d\vec{l} \times \vec{B}$$

$$\vec{F}_B = ILB\sin 90^\circ(\cos\theta\,\hat{i} - \sin\theta\,\hat{k})$$

$$\vec{\tau}_B + \vec{\tau}_s = 0$$

$$\vec{r}_1 \times \vec{F}_B + \vec{r}_2 \times \vec{F}_s = 0$$

$$\frac{L}{2} F_B\,\hat{j} + L F_s \sin(\pi-\theta)(-\hat{j}) = 0$$



$$\frac{x}{L} = \cos\theta$$

$$x = L\cos\theta$$

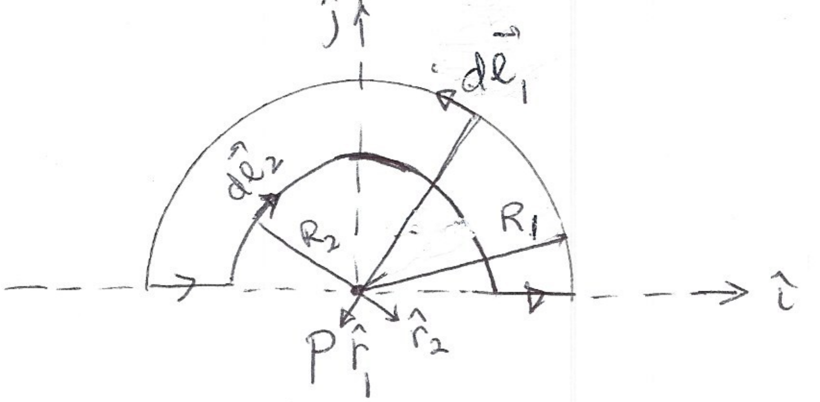
$$\frac{\cancel{L}}{2}(ILB) - \cancel{L}kx\sin\theta = 0$$

$$\frac{I\cancel{L}B}{2} = k\cancel{L}\cos\theta\sin\theta$$

$$B = \frac{2k\cos\theta\sin\theta}{I}$$

$$\boxed{\vec{B} = \frac{k}{I}\sin 2\theta\,\hat{j}}$$

2. The wire shown is bent into two semicircular arcs of radius R_1 and R_2 and share a common center at P. The wire carries current I with the direction indicated. Starting with the Biot Savart Law, find the net magnetic field at point P, the center of the two arcs.



$$\vec{B} = \frac{\mu_0}{4\pi}\left[\int_0^{\pi R_1} \frac{I\, d\vec{l}_1 \times \hat{r}_1}{R_1^2} + \int_{R_1}^{R_2} \cancel{\frac{I\, d\vec{l}_3 \times \hat{r}_3}{r_3^2}}^{\,0} + \int_0^{\pi R_2} \frac{I\, d\vec{l}_2 \times \hat{r}_2}{R_2^2} + \int_{R_2}^{R_1} \cancel{\frac{I\, d\vec{l}_4 \times \hat{r}_4}{r_4^2}}^{\,0}\right]$$

$d\vec{l}_3 \parallel \hat{r}_3$

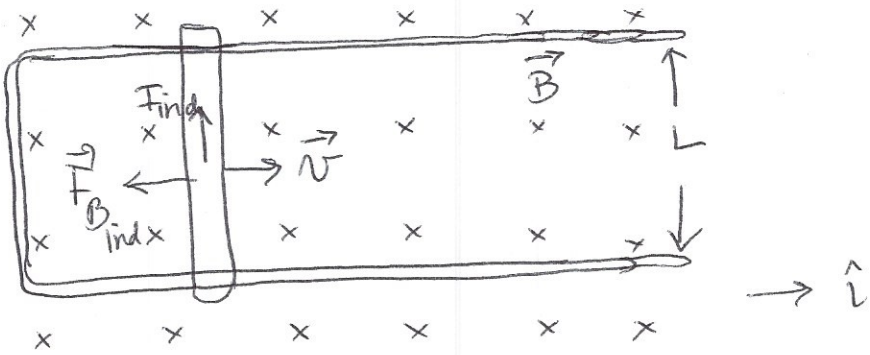
$d\vec{l}_4 \parallel \hat{r}_4$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\left[\int_0^{\pi R_1} \frac{dl_1 \sin 90^\circ\, \hat{k}}{R_1^2} + \int_0^{\pi R_2} \frac{dl_2 \sin 90\, (-\hat{k})}{R_2^2}\right]$$

$$\vec{B} = \frac{\mu_0 I}{4\cancel{\pi}}\left[\frac{\cancel{\pi} \cancel{R_1}\, \hat{k}}{R_1^{\cancel{2}}} + \frac{\cancel{\pi}\cancel{R_2}}{R_2^{\cancel{2}}}(-\hat{k})\right]$$

$$\boxed{\vec{B} = \frac{\mu_0 I\, \hat{k}}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

3. A rectangular loop with width L and a sliding bar of mass m and resistance R is shown in the diagram. A uniform magnetic field $\boldsymbol{B}$ is directed into the page as shown. The bar is given an initial speed of V_o and then released. There is no friction between the bar and the loop and the resistance of the loop is negligible in comparison to the resistance R of the sliding bar. Find the distance the bar moves before coming to rest.



$$\mathcal{E}_{ind} = I_{ind} R = -\frac{d\Phi_m}{dt}$$

$$I_{ind} = \frac{1}{R}\left(-\frac{d}{dt}\int \vec{B}\cdot\hat{n}\,dA\right) = \frac{1}{R}\left(-\frac{d}{dt} BLx\right)$$

$$I_{ind} = -\frac{BL}{R}\frac{dx}{dt}$$

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F}_{B_{ind}} = m\vec{a}$$

$$I_{ind}\vec{\ell}\times\vec{B} = m\vec{a}$$

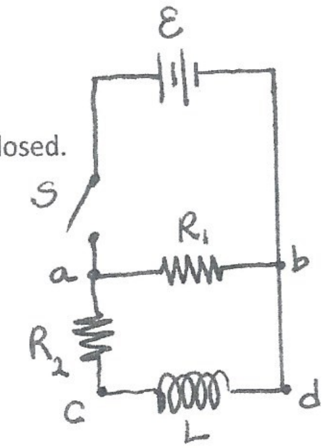
$$I_{ind}LB\cancel{(-\hat{\imath})} = ma\cancel{(-\hat{\imath})}$$

$$-\frac{B^2L^2}{R}\frac{dx}{\cancel{dt}} = \frac{m\,dv}{\cancel{dt}}$$

$$\int_0^{x_f} -\frac{B^2L^2}{R}\,dx = m\int_{V_0}^{0} dv$$

$$x_f = \frac{-R}{B^2L^2}m(0-v_0) = \frac{mRv_0}{B^2L^2}$$

4. Consider the circuit shown.



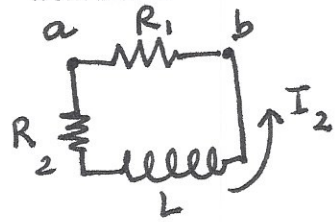
a. (5 points) Find the potential differences V_{ab} and V_{cd} the instant the switch is closed.

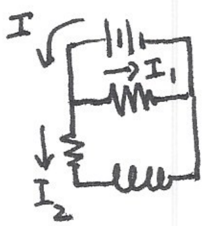
$\boxed{V_{ab} = \varepsilon}$ → $\varepsilon - V_R = 0 \quad V_R = V_{ab}$

$\boxed{V_{cd} = \varepsilon}$ → $\varepsilon - \cancel{I_2 R_2}^{\,0} - V_L = 0$

$V_L = \varepsilon = V_{cd}$

b. (10 points) The switch has been closed for a long time and steady state currents are established. Suddenly the switch is opened. What is the current through R_1 and in which direction, from a to b or from b to a?





$\varepsilon - I_1 R_1 = 0$

$\varepsilon - I_2 R_2 = 0$ → $L \cancel{\frac{dI}{dt}}^{\,0}$

$$\boxed{I_2 = \frac{\varepsilon}{R_2}}$$

c. (10 points) Let the instant the switch is opened be time $t = 0$. Starting with Kirchoff's Loop Equation, derive an expression for the current in R_1 as a function of time.

$$-V_L - IR_1 - IR_2 = 0$$

$$L\frac{dI}{dt} = -I(R_1 + R_2)$$

$$\int_{I_2}^{I} \frac{dI}{I} = -\int_0^t \frac{(R_1+R_2)}{L}\,dt$$

$$\ln\frac{I}{I_2} = -\frac{(R_1+R_2)}{L}t$$

$$I = I_2 e^{-\left(\frac{R_1+R_2}{L}\right)t}$$

$$\boxed{I = \frac{\varepsilon}{R_2} e^{-\frac{(R_1+R_2)t}{L}}}$$