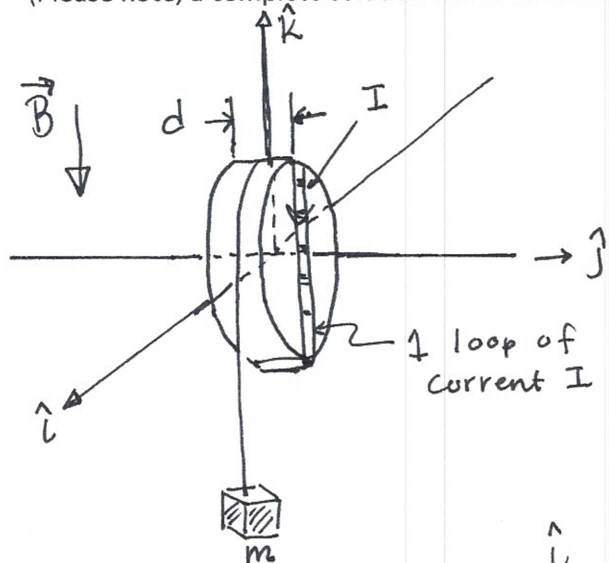
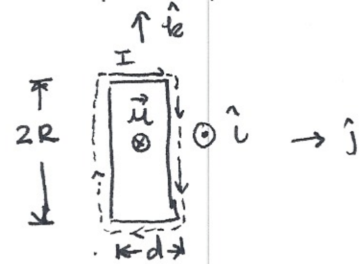
4B Winter 2018 Exam 2 Name SOLUTION

Show all your work for full credit. You have one (1) hour for the four questions. No note cards, scratch papers, cell phones, calculators, or other electronic devices are allowed.

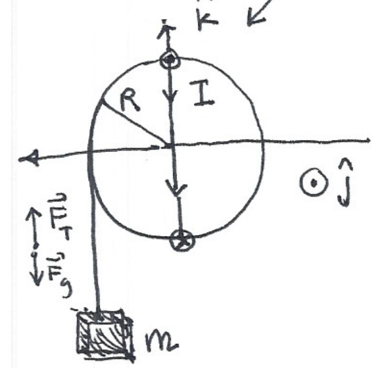
1. A single loop, rectangular coil of current, I, is wrapped around the diameter of a cylindrical drum of radius R and thickness d that is free to rotate about the y-axis as shown in the diagram. A massless string is wound around the drum and a mass m is attached. Gravity is present. **Find the value of the current, I, so that the drum does not rotate.** (Please note, a complete solution will include the vector notation as given in the problem.)





$\vec{\mu} = NIA\,\hat{n}$

$\vec{\mu} = (1)(I)(2R\times d)(-\hat{i})$



system: drum

$\vec{\tau}_{net} = I\cancel{\vec{\alpha}}^{\,0}$

$\vec{\tau}_T + \vec{\tau}_B = 0$

$\vec{r}\times\vec{F}_T + \vec{\mu}\times\vec{B} = 0$

$\left(\cancel{R}mg\hat{j} - 2\cancel{R}dIB\hat{j} = 0\right)\cdot\hat{j}$

$2IdB = mg$

$$\boxed{I = \frac{mg}{2dB}}$$

system: mass

$\vec{F}_{net} = m\cancel{\vec{a}}^{\,0}$

$\vec{F}_T + \vec{F}_g = 0$

$\hat{k}:\ F_T - F_g = 0$

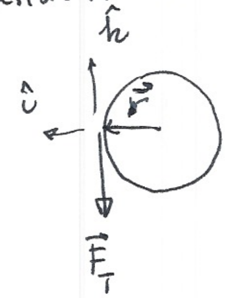
$F_T = mg$

(ideal string so also the magnitude of the tension at the drum)

$\vec{\tau}_B = \vec{\mu}\times\vec{B}$

$$\vec{\tau}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -I2Rd & 0 & 0 \\ 0 & 0 & -B \end{vmatrix}$$

$\vec{\tau}_B = -2RdIB\,\hat{j}$

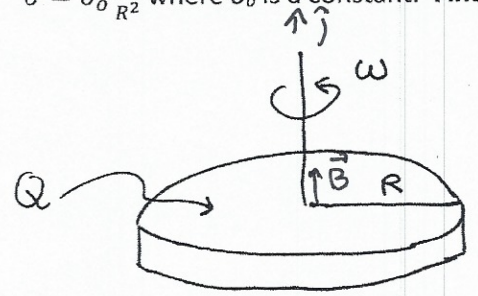


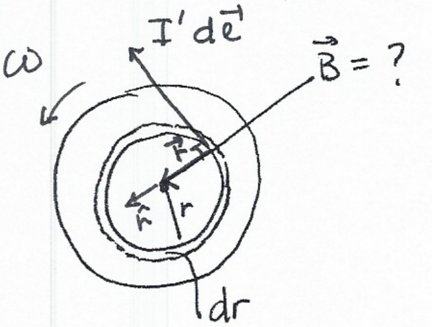
$\vec{\tau}_T = \vec{r}\times\vec{F}_T$

$$\vec{\tau}_T = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ R & 0 & 0 \\ 0 & 0 & -mg \end{vmatrix}$$

$\vec{\tau}_T = Rmg\,\hat{j}$

2. A dielectric disk has radius, R, total positive charge Q and is spinning about and axis through its center with angular velocity ω as shown. The positive charge distribution is not constant, but varies with r according to:
$\sigma = \sigma_o \frac{r^2}{R^2}$ where σ_o is a constant. **Find the magnetic field, $\vec{B}$, at the center of the spinning disk of charge.**





here, all rings, regardless of r, have period T, where $T = \frac{2\pi}{\omega}$

$\vec{B} = \int d\vec{B}$ (p.o.s.)

let B' be the field due to the ring at r, width dr. Call that current I'. Then replace $B' \to dB$ and $I' \to dI$

$$B'_{\text{(one ring)}} = \frac{\mu_o}{4\pi}\int \frac{I'\, d\vec{l} \times \hat{r}}{r^2} \qquad d\vec{l} \perp \hat{r} \qquad |d\vec{l}| = r\,d\theta$$

$$B'_{\text{(one ring)}} = \frac{\mu_o}{4\pi}\int_0^{2\pi} \frac{I' \cancel{r}\, d\theta \sin 90°}{r^{\cancel{2}}}$$

$$B'_{\text{(one ring)}} = \frac{\mu_o I' \cancel{2\pi}}{2\cancel{4\pi}\, r}$$

$$\frac{dQ}{dA} = \sigma$$

$$dQ = \sigma\, dA$$

$$dI = \frac{dQ}{T} = \frac{\sigma\, dA \cdot \omega}{2\pi}$$

$$Q = \int_0^R \frac{\sigma_o r^2 2\pi r\, dr}{R^2}$$

$$dI = \frac{\sigma_o r^2 \cdot \cancel{2\pi} r\, dr\, \omega}{R^2\ \cancel{2\pi}}$$

$$Q = \frac{\sigma_o 2\pi}{R^2}\int_0^R r^3 dr$$

$$dI = \frac{\sigma_o \omega r^3 dr}{R^2}$$

$$Q = \frac{\sigma_o \cancel{2}\pi}{\cancel{R^2}} \frac{R^{\cancel{4}\,2}}{\cancel{4}\,2}$$

$$dB = \frac{\mu_o dI}{2r}$$

$$Q = \frac{\pi \sigma_o R^2}{2}$$

$$B = \int_0^R \frac{\mu_o \sigma_o \omega r^{\cancel{3}\,2} dr}{R^2\, 2\cancel{r}}$$

$$\sigma_o R = \frac{2Q}{\pi R}$$

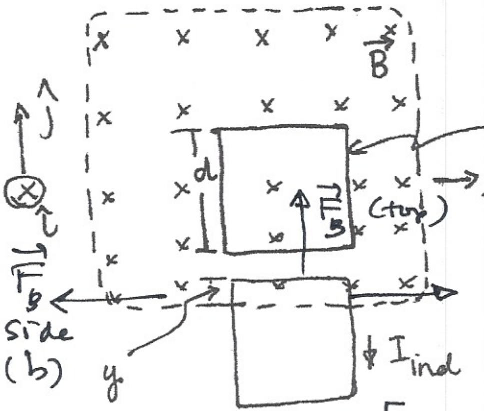
$$B = \frac{\mu_o \sigma_o \omega}{2R^2} \left.\frac{r^3}{3}\right|_0^R = \frac{\mu_o \sigma_o \omega R^{\cancel{3}}}{6 \cancel{R^2}} \to \frac{\mu_o \omega}{6} \cdot \sigma_o R$$

$$\boxed{\vec{B} = \frac{\mu_o \omega Q}{3\pi R}\,\hat{j}}$$

$$\mu_o \frac{\cancel{2}Q}{\pi R} \cdot \frac{\omega}{\cancel{6}\,3}$$

3. A vertically oriented, square loop of copper wire falls from a region where the magnetic field $\vec{B}$ is horizontal, uniform, and perpendicular to the plane of the loop, into a region where the field is zero. The loop is released from rest and initially is entirely within the magnetic field region. Let the side of the loop be *d*, the total resistance *R*, and the total mass *M*. If the loop reaches terminal speed while the upper segment is still in the magnetic field region, **find an expression for the speed as function of time and use that to determine the terminal speed.**

(DO NOT IGNORE GRAVITY!)



mass m
Resistance R
side d

$\vec{F}_B$ side (a)

[The $\Sigma \vec{F}_B$ side(a) + (b) = 0]

System: Loop

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F}_B + \vec{F}_g = m\vec{a}$$

$$IdB - Mg = -Ma$$

$$\frac{B^2 d^2 v}{R} - Mg = -M\frac{dv}{dt}$$

$$\boxed{V - \frac{RMg}{B^2d^2} = -\frac{RM}{B^2d^2}\frac{dv}{dt}} = 0 \quad \text{for } V_T$$

$$\int_{V_1}^{V} \frac{dv}{V - \frac{RMg}{B^2d^2}} = \int_{t_1}^{t} -\frac{B^2d^2}{RM}\,dt$$

$$\ln\left(\frac{V - RMg/B^2d^2}{V_1 - RMg/B^2d^2}\right) = -\frac{B^2d^2}{RM}(t - t_1)$$

$$V - \frac{RMg}{B^2d^2} = \left(V_1 - \frac{RMg}{B^2d^2}\right)e^{-\frac{B^2d^2}{RM}(t - \frac{V_1}{g})}$$

$$V = \frac{RMg}{B^2d^2}\left(1 - e^{-B^2d^2/RM\,(t - V_1/g)}\right) + V_1 e^{-\frac{B^2d^2}{RM}(t - V_1/g)}$$

some constant

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d}{dt}\int \vec{B}\cdot\hat{n}\,da$$

here $\vec{B} \parallel \hat{n}$

$$\mathcal{E} = -\frac{d}{dt}Bdy$$

$$\mathcal{E} = -Bdv$$

$$I = \frac{|\mathcal{E}|}{R} = \frac{Bdv}{R}$$

$$\vec{F}_B = \int I\,d\vec{\ell}\times\vec{B} \qquad d\vec{\ell} = dz\,\hat{k}$$

$$\vec{F}_B = IdB\,\hat{j} \qquad \int_0^d dz = d$$

$$V_{1y} = \cancel{V_{0y}}^{\,0} + a_y t$$

$$-V_1 = -gt_1$$

$$V_1 = gt_1$$

my picture has the loop a bit above the field So, it is in free fall for a time t.

$$\boxed{V_T = \frac{RMg}{B^2d^2}}$$

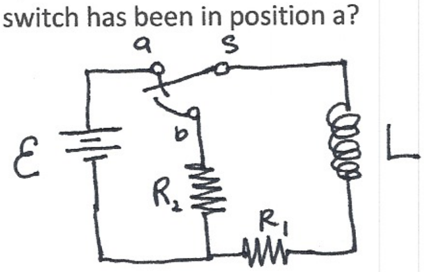
same

$t \to \infty$

$$\boxed{V_T = \frac{RMg}{B^2d^2}}$$

4. The *RL* circuit shown can be used to generate time-varying high voltage from a low-voltage source. $R_2 = c\,R_1$, where c is a large positive constant.

a) The switch is put to position a and is there for a long time. What is the steady state current, *I*, a long time after the switch has been in position a?

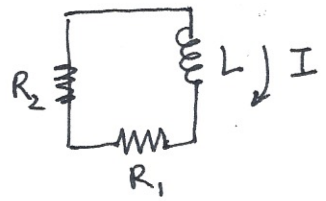


$$\mathcal{E} - L\frac{dI}{dt} - IR_1 = 0$$

steady state $\rightarrow \frac{dI}{dt} \rightarrow 0$

$$I = \frac{\mathcal{E}}{R_1}$$

b) The switch is thrown quickly to position b. Find the current through resistor R_1 as a function of time.



$$-L\frac{dI}{dt} - IR_1 - IR_2 = 0$$

$$\frac{-L}{R_1+R_2}\left(\frac{dI}{dt}\right) = I$$

$$\int_{I_i}^{I} \frac{dI}{I} = -\int_0^t \frac{R_1+R_2}{L}\,dt$$

$$\ln\frac{I}{I_i} = \frac{-(R_1+R_2)\,t}{L}$$

$$I = I_i\, e^{-\frac{(R_1+R_2)}{L}t}$$

$$\boxed{I = \frac{\mathcal{E}}{R_1} e^{-\frac{(R_1+R_2)}{L}t}} = \frac{\mathcal{E}}{R_1} e^{-\frac{R_1(1+c)}{L}t}$$

c) Compute the initial voltage across each resistor and across the inductor, the instant the switch is moved to b.

$$V_{R2} = I_i R_2 = \frac{\mathcal{E}}{R_1} R_2 = \boxed{\mathcal{E}c}$$

$$V_{R1} = I_i R_1 = \frac{\mathcal{E}}{R_1} R_1 = \mathcal{E}$$

$$V_L + V_{R1} + V_{R2} = 0$$

$$\boxed{V_L = -\mathcal{E}c - \mathcal{E} = -\mathcal{E}(1+c)}$$