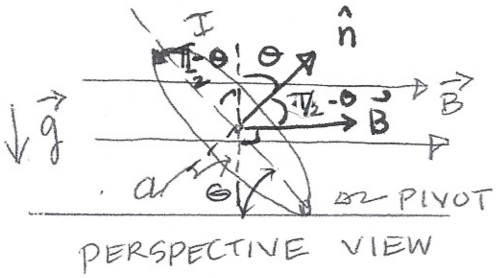
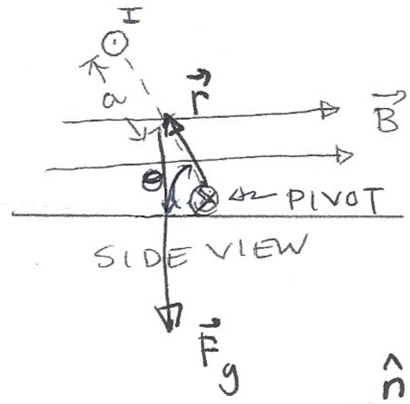
Physics 4B Winter 2019 - EXAM 2
Dickson

Name SOLUTIONS

Show all your work for full credit. No calculators, note cards, scratch paper or electronic devices are allowed during the exam. Complete all four questions. You have one hour.

1. A single loop of wire has mass *m* and radius *a*. The loop is in a uniform, horizontal magnetic field B as shown. ~~A flat, compact loop of wire with n turns wrapped around it, with each turn concentric with the sphere.~~ Gravity is present. **Find the value of the current, I, in the loop so that the loop makes an angle θ with the horizontal.** The point of contact with the ground is a pivot, which holds the loop in place.

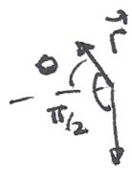




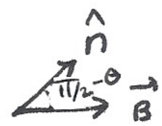
System: Loop

$\vec\tau_{net} = I\cancel{\vec\alpha}^{\,0}$

$\vec\tau_g + \vec\tau_B = 0$



$\vec\tau_g$ ⊙ out



⊗ $\vec\tau_B$ in

let out be (+)

$\vec r \times \vec F_g + \vec\mu \times \vec B = 0$

$$+|\vec r||\vec F_g|\sin(\pi/2+\theta) - nIA|\hat n||\vec B|\sin(\pi/2-\theta) = 0$$

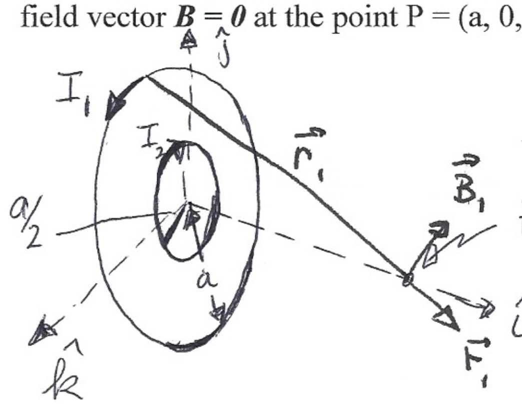
$\sin(\pi/2+\theta) = \sin(\pi/2-\theta) = \cos\theta$; $n=1$; $A=\pi a^2$, $r=a$

$$\cancel{a}mg\cancel{\cos\theta} - (1)I\pi a^{\cancel{2}}B\cancel{\cos\theta} = 0$$

$$I\pi aB = mg$$

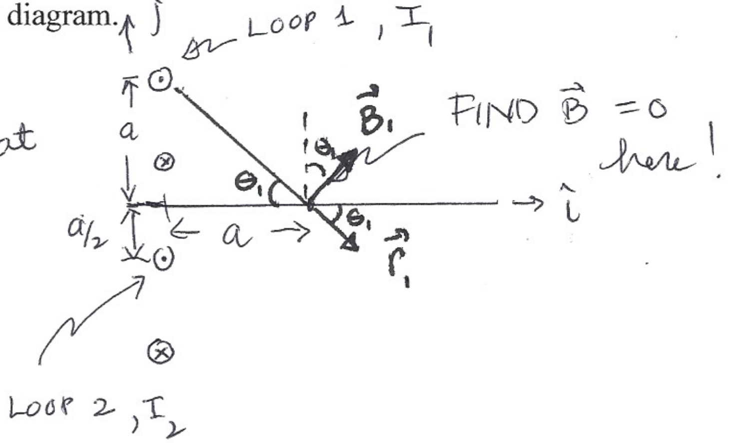
$$\boxed{I = \frac{mg}{\pi a B}}$$

2. Two circular loops of wire are in the y-z plane, both centered at $x = 0$. Loop 1 carries current I_1 and has radius a. Loop 2 carries a current in the opposite sense and has radius $^a/_2$. **Find the current I_2**, so that the magnetic field vector $\boldsymbol{B} = \boldsymbol{0}$ at the point P = (a, 0, 0) as shown in the diagram.



Find $\vec{B} = 0$ at $P = (a, 0, 0)$

PERSPECTIVE VIEW



$$\vec{B}_1 = \frac{\mu_0}{4\pi} \int \frac{I_1 \, d\vec{\ell}_1 \times \hat{r}_1}{r_1^2}$$

by symmetry, the x-components remain
y-component $\int \to 0$

$$B_{1x} = \frac{\mu_0 I_1}{4\pi} \int_0^{2\pi a} \frac{d\ell_1 \sin\theta_1}{r_1^2}$$

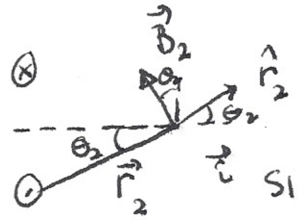
$d\vec{\ell}_1 \perp \hat{r}_1$ $\qquad B_{1x} = B_1 \sin\theta_1 \hat{i}$

$$\sin\theta_1 = \frac{a}{\sqrt{a^2 + a^2}} = \frac{\cancel{a}}{\sqrt{2}\cancel{a}} = \frac{1}{\sqrt{2}}$$

$$B_{1x} = \frac{\mu_0 I_1}{4\cancel{\pi}} \frac{1}{\sqrt{2}} \frac{1}{\cancel{2}a^{\cancel{2}}} \cancel{2\pi a}$$

$$r_1^2 = \left(\sqrt{a^2 + a^2}\right)^2 = 2a^2$$

$$B_{1x} = \frac{\sqrt{2}\mu_0 I_1}{8a}$$



$$\vec{B}_2 = B_{2x}\hat{i} + \cancel{B_{2y}\hat{j}}^{\,0}$$

$$\vec{B}_2 = \frac{\mu_0}{4\pi} \int \frac{I_2 \, d\vec{\ell}_2 \times \hat{r}_2}{r_2^2}$$

$$\sin(\pi/2 + \theta_2) = \sin\theta_2$$

by symmetry only the x-components remain $\quad$ y-components $\int \Rightarrow 0$

$$B_{2x} = \frac{\mu_0 I_2}{4\pi} \int_0^{\pi a} \frac{d\ell_2 \sin(\pi/2 + \theta_2)}{r_2^2}$$

$d\vec{\ell}_2 \perp \hat{r}_2$

$$\sin\theta_2 = \frac{a/2}{\sqrt{(a/2)^2 + a^2}} = \frac{a/2}{\sqrt{5}\, a/2}$$

$$B_{2x} = -\frac{\mu_0 I_2}{4\pi} \int^{\pi a} \frac{d\ell_2 \sin(\theta_2)}{\frac{5}{4} a^2}$$

$$\sin\theta_2 = \frac{1}{\sqrt{5}}$$

$$r_2^2 = \left(\left(\frac{a}{2}\right)^2 + a^2\right) = \frac{5}{4}a^2$$

$$B_{2x} = -\frac{\mu_0 I_2}{5\pi a^{\cancel{2}}} \frac{1}{\sqrt{5}} \pi \cancel{a} =$$

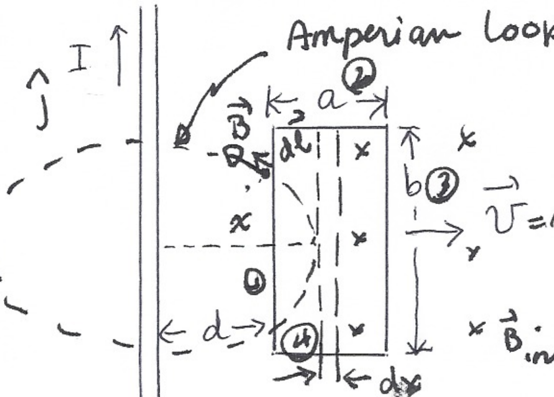
$$\vec{B}_{net} = \vec{B}_1 + \vec{B}_2 = 0$$

$$\left[0 = \frac{\sqrt{2}\, \cancel{\mu_0} I_1 \hat{i}}{8\cancel{a}} - \frac{\cancel{\mu_0} I_2}{5\sqrt{5}\cancel{a}} \hat{i} \right] \cdot \hat{i}$$

$$\frac{I_2}{5\sqrt{5}} = \frac{\sqrt{2}\, I_1}{8}$$

$$\boxed{I_2 = \frac{5\sqrt{10}}{8} I_1}$$

3. A rectangular loop of wire has width a and length b as shown. It is near an infinitely long current carrying wire with the long side, b, parallel to the wire. The wire carries current I in the senses shown in the diagram. The loop is pulled to the right with constant velocity v. At time $t = 0$, the left side of the loop is distance d from the long straight wire. **Find the net induced EMF, $\mathcal{E}$, in the loop as a function of time.**



at $t=0$

① Find $\vec{B}$ in the region of the Loop using A.L

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc} \qquad \{\vec{B} \parallel d\vec{\ell}\}$$

$$\oint B\,d\ell = \mu_0 I_{enc} \qquad \{B \text{ is uniform at distance } x \text{ away}$$

$$B\oint d\ell = \mu_0 I_{enc}$$

$$B\,2\pi x = \mu_0 I_{enc}$$

$$B = \frac{\mu_0 I_{enc}}{2\pi x}\ ; \quad \vec{B} = \frac{\mu_0 I}{2\pi x}(-\hat{k})$$

here note: $x = d + vt$

Method 1

Using motional EMF for the 4 segments

$$\mathcal{E}_1 = \int_0^b (\vec{v}\times\vec{B})\cdot d\vec{\ell}_1$$

$$\mathcal{E}_1 = \int_0^b \frac{v\,\mu_0 I}{2\pi(d+vt)}\,\hat{j}\cdot d\ell_1\hat{j}$$

$$\mathcal{E}_1 = \frac{\mu_0 I b v}{2\pi(d+vt)}$$

$$\mathcal{E}_2 = \int(\vec{v}\times\vec{B})\cdot d\vec{\ell}_2 = \mathcal{E}_4$$

$$= \int vB\,\cancel{\hat{j}\cdot d\ell_2\hat{i}}^{\,0}$$

$$\mathcal{E}_3 = +\int_0^b(\vec{v}\times\vec{B})\cdot d\vec{\ell}_3$$

here ; $x = d + a + vt$

$$\mathcal{E}_3 = \int_0^b \frac{v\mu_0 I}{2\pi(d+a+vt)}\,\hat{j}\cdot d\ell\hat{j}$$

$$\mathcal{E}_3 = \frac{\mu_0 I b v}{2\pi(d+a+vt)}$$

$$\mathcal{E}_{TOTAL} = \mathcal{E}_1 - \mathcal{E}_3$$

$$\boxed{\mathcal{E}_{TOTAL} = \frac{\mu_0 I b v}{2\pi}\left(\frac{1}{d+vt} - \frac{1}{d+a+vt}\right)}$$

both EMFs are ⊕ higher V at the top, so the contributions subtract

Method 2 $\mathcal{E} = -\dfrac{d\Phi}{dt}$

Find the flux and take the derivative w.r.t to

Let $(\vec{B} \parallel \hat{n})$

$$\Phi_B = \int \vec{B}\cdot\hat{n}\,dA$$

$$\Phi_B = \frac{\mu_0 I}{2\pi}\int_{d+vt}^{d+a+vt}\frac{b\,dx}{x}$$

$$\Phi_B = \frac{\mu_0 I b}{2\pi}\ln\left(\frac{d+a+vt}{d+vt}\right)$$

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

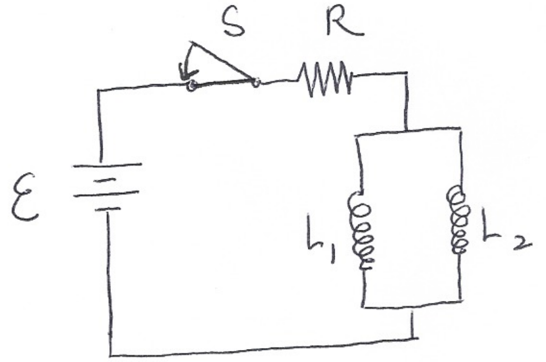
$$\mathcal{E} = -\frac{d}{dt}\left[\frac{\mu_0 I b}{2\pi}\left(\ln(d+a+vt) - \ln(d+vt)\right)\right]$$

$$\boxed{\mathcal{E} = \frac{\mu_0 I b v}{2\pi}\left(\frac{1}{d+vt} - \frac{1}{d+a+vt}\right)}$$

same but faster method

4. A circuit consists of an EMF $\mathcal{E}$, a switch S, a resistor R and two inductors L_1 and L_2 as shown in the following diagram. Please assume that the separation distance between the two inductors is large enough so that the mutual inductance can be ignored.

Find I_1 and I_2, the currents through each inductor L_1 and L_2 as functions of time.



① $\mathcal{E} - IR - L_1 \frac{dI_1}{dt} = 0$

② $\mathcal{E} - IR - L_2 \frac{dI_2}{dt} = 0$

③ $I = I_1 + I_2$

3' $\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$

①-③ = $-L_1 \frac{dI_1}{dt} + L_2 \frac{dI_2}{dt} = 0$

$\frac{dI_1}{dt} = \frac{L_2}{L_1} \frac{dI_2}{dt}$ in 3'

$I_1 = \frac{L_2 I_2}{L_1}$ in ③

$\frac{dI}{dt} = \frac{L_2}{L_1}\frac{dI_2}{dt} + \frac{dI_2}{dt}$

$\frac{dI}{dt} = \frac{L_2 + L_1}{L_1} \frac{dI_2}{dt}$

$\frac{dI_2}{dt} = \frac{L_1}{L_2 + L_1} \frac{dI}{dt}$ in ②

$\mathcal{E} - IR - \frac{L_1 L_2}{L_2 + L_1} \frac{dI}{dt} = 0$

$\frac{\mathcal{E}}{R} - I - \frac{L_1 L_2}{R(L_2 + L_1)} \frac{dI}{dt} = 0$

$\frac{-L_1 L_2}{R(L_1 + L_2)} \frac{dI}{dt} = I - \mathcal{E}/R$

$\int_0^I \frac{dI}{I - \mathcal{E}/R} = \int_0^t -\frac{R(L_1 + L_2)}{L_1 L_2} dt$

$\ln \frac{I - \mathcal{E}/R}{-\mathcal{E}/R} = -\frac{R(L_1 + L_2)}{L_1 L_2} t$

$I(t) = \mathcal{E}/R \left(1 - e^{-\frac{R(L_1 + L_2)}{L_1 L_2} t}\right)$

$I = \frac{L_2}{L_1} I_2 + I_2$

$I = \frac{(L_1 + L_2) I_2}{L_1}$

$I_2 = \frac{I L_1}{L_1 + L_2}$

$$\boxed{I_2 = \frac{\mathcal{E}}{R} \frac{L_1}{(L_1 + L_2)} \left(1 - e^{-\frac{R(L_1 + L_2)}{L_1 L_2} t}\right)}$$

$I_1 = \frac{L_2}{L_1} I_2$

$$\boxed{I_1 = \frac{\mathcal{E} L_2}{R(L_1 + L_2)} \left(1 - e^{-\frac{R(L_1 + L_2)}{L_1 L_2} t}\right)}$$