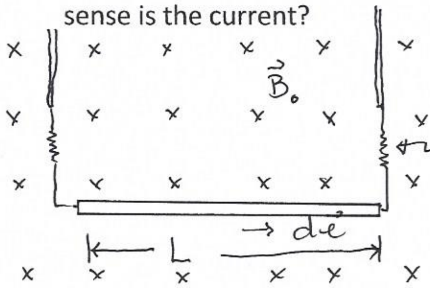
4B Exam 2 -- March 17th, 2020 Name SOLUTION

Winter Quarter CWID

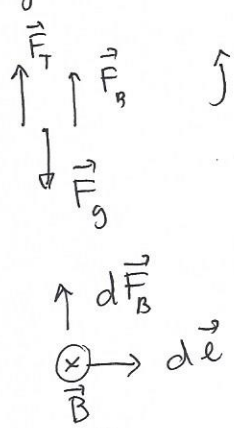
Show all your (own) work for full credit. You may refer to the enclosed equation sheet during the exam; otherwise, the exam is closed book, closed notes. Due to the extenuating circumstances, the exam time is extended to 90 minutes for four (4) questions. **The exam start time is 5:30 pm and must be submitted via email no later than 7:00 pm.** PDF or jpeg format, please.

1. A wire of length *L* and mass *M* is suspended in a magnetic field of $\boldsymbol{B_o}$ into the page as shown. What are the magnitude of the current required to remove the tension from the supporting rods? In which sense is the current?



NOT SPRINGS OR RESISTORS JUST WIRES THAT HAVE TENSION

System: wire



must be this way!

$$\vec{F}_{net} = m\cancel{\vec{a}}^{\,0}$$

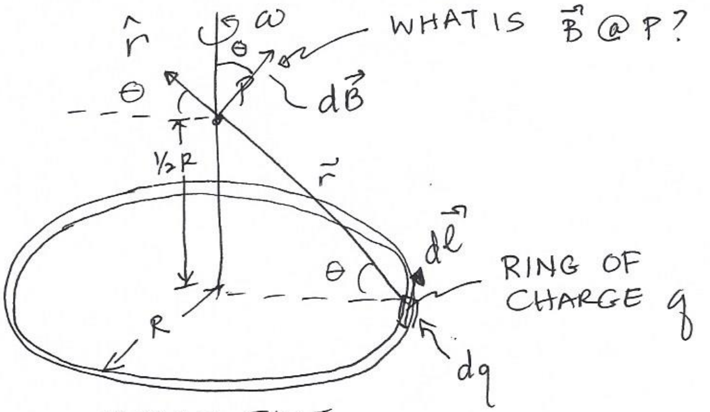
$$\cancel{\vec{F}_T} + \vec{F}_B + \vec{F}_g = 0$$

$$\int_0^L I\, d\vec{\ell} \times \vec{B}_o + mg(-\hat{j}) = 0$$

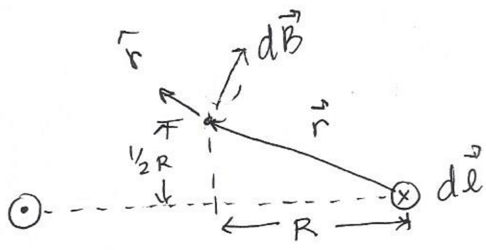
$$ILB_o \sin 90(\hat{j}) + mg(-\hat{j}) = 0$$

$$\boxed{I = \frac{mg}{LB_o}}$$

2. A nonconducting ring of radius R is uniformly charged with a total positive charge q. The ring rotates at a constant angular speed ω about an axis through its center, perpendicular to the plane of the ring. What is the magnitude of the magnetic field on the axis of the ring a distance ½ R from its center?



PERSPECTIVE VIEW



CROSS-SECTION VIEW

$$I = \frac{dq}{dt} \rightsquigarrow dq = I\,dt$$

$$Q = \int_0^T I\,dt = IT$$

$$\omega = 2\pi f = \frac{2\pi}{T} \rightsquigarrow \frac{1}{T} = \frac{\omega}{2\pi}$$

$$I = \frac{Q}{T} = \frac{\omega Q}{2\pi} \quad \text{constant}$$

$$V = R\omega$$

$$\frac{d\ell}{dt} = V$$

$$d\ell = R\omega\,dt$$

always ⊥

$$d\vec{\ell} \times \hat{r} = |d\vec{\ell}||\hat{r}| \sin 90°$$

$$\vec{B} = \int d\vec{B}$$

$$B = \int dB \cos\theta$$

$$\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\,d\vec{\ell} \times \hat{r}}{r^2}$$

by symmetry, as you go around the ring only the vertical component survives.
By P.O.S. the component parallel to the ring $\int \to 0$

$$B = \frac{\mu_0 I}{4\pi} \int_0^{2\pi R} \frac{d\ell \cos\theta}{r^2}$$

$$\cos\theta = \frac{R}{r} = \frac{R}{\sqrt{R^2 + (\frac{1}{2}R)^2}}$$

$$\cos\theta = \frac{R}{\sqrt{R^2(1+\frac{1}{4})}} = \frac{\cancel{R}}{\cancel{R}\sqrt{\frac{5}{4}}}$$

$$\cos\theta = \frac{2\sqrt{5}}{5}$$

$$B = \frac{\mu_0 \omega Q}{(4\pi)(2\pi)} \int_0^{2\pi R} \frac{d\ell}{R^2\left(\frac{5}{4}\right)} \frac{2\sqrt{5}}{5}$$

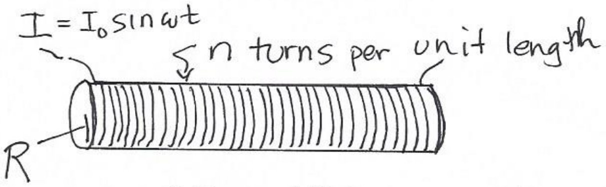
$$B = \frac{\mu_0 \omega Q}{4\pi\,(2\pi)} \left[\frac{(2\sqrt{5})(4)}{25\,R^2}\, 2\pi R \right]$$

$$\boxed{B = \frac{2\sqrt{5}\,\mu_0 \omega Q}{25\pi R}}$$

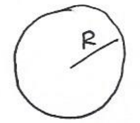
$$r^2 = \left(R^2 + (\tfrac{1}{2}R)^2\right) = R^2(1+\tfrac{1}{4})$$

$$r^2 = R^2\left(\frac{5}{4}\right)$$

3. A long solenoid has n turns per unit length and carries a current given by $I = I_0 \sin \omega t$. The solenoid has a circular cross section of radius R. Find the induced electric field at a radius r from the axis of the solenoid for $r < R$ and $r > R$.



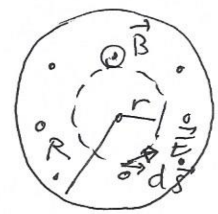
SIDE VIEW



LARGER END OR CROSS-SECTION VIEW

$d\vec{\ell}$ or $d\vec{s}$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$



For $r < R$

$$\oint E\, ds \cos 0^\circ = -\frac{d}{dt}\int \vec{B}\cdot\hat{n}\, dA$$

$B = \mu_0 n I$ for a solenoid

$$E\, 2\pi r = -\frac{d}{dt}\int \mu_0 n I\, dA$$

Area $= \pi r^2$

$$(E)(2\pi r) = -\frac{d}{dt}\left(\mu_0 n I_0 \sin \omega t\, A\right)$$

$$E(r<R) = -\frac{\mu_0 n \cancel{\pi} r^{\cancel{2}}}{2\cancel{\pi}\cancel{r}}\, \omega I_0 \cos \omega t$$

$$\boxed{E = -\frac{\mu_0 n \omega r I_0 \cos \omega t}{2} \quad \{r<R\}}$$

for $r > R$

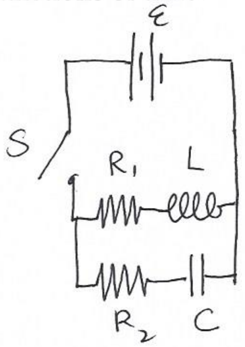
Area $= \pi R^2$

$$E(2\pi r) = -\frac{d}{dt}\left(\mu_0 n I_0 \sin \omega t\, A\right)$$

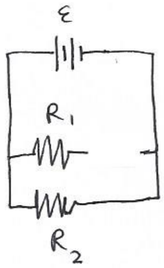
$$E = \frac{-\mu_0 n \omega \cancel{\pi} R^2 I_0 \cos \omega t}{2\cancel{\pi} r}$$

$$\boxed{E = \frac{-\mu_0 n \omega R^2 I_0 \cos \omega t}{2r} \quad \{r>R\}}$$

4. In the circuit shown, the capacitor is initially uncharged. The ε, L, R_1, R_2, and C are known. At time $t = 0$, the switch is closed. After a long time, the charge on the capacitor is Q_{max}. What are I_1, I_2, and Q as functions of time?

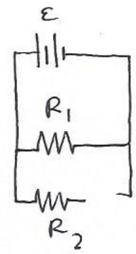


at $t = 0$



$$\varepsilon - I_i R_2 = 0$$

at $t = \infty$



$$\varepsilon - I_f R_1 = 0$$

$$\frac{Q_{max}}{C} = V_{cap\ final} = \varepsilon$$

$$Q_{max} = \varepsilon C$$

$$\varepsilon - I_1 R_1 - L\frac{dI_1}{dt} = 0$$

$$\varepsilon - I_2 R_2 - \frac{Q_2}{C} = 0$$

$$\varepsilon - \frac{dQ_2}{dt} R_2 - \frac{Q_2}{C} = 0$$

$$\frac{Q_2}{R_2 C} - \varepsilon \cancel{C} = -\frac{dQ_2}{dt}$$

$$\int \frac{dQ_2}{Q_2 - \varepsilon C} = \int -\frac{1}{R_2 C}\, dt$$

$$\ln\left|\frac{Q_2 - \varepsilon C}{-\varepsilon C}\right| = -\frac{1}{R_2 C} t$$

$$\boxed{Q_2 = \varepsilon C\left(1 - e^{-t/\tau_c}\right) \qquad \tau_c = R_2 C}$$

$$\boxed{I_2 = \frac{dQ_2}{dt} = \frac{\varepsilon \cancel{C}}{R_2 \cancel{C}} e^{-t/\tau_c}}$$

$$\frac{\varepsilon}{R_1} - I_1 - \frac{L}{R_1}\frac{dI_1}{dt} = 0$$

$$-\frac{L}{R_1}\frac{dI_1}{dt} = I_1 - \frac{\varepsilon}{R_1}$$

$$\int_0^{I_1} \frac{dI_1'}{I_1' - \varepsilon/R_1} = \int_0^t -\frac{R_1}{L}\, dt'$$

$$\ln\left|\frac{I_1 - \varepsilon/R_1}{-\varepsilon/R_1}\right| = \frac{-R_1 t}{L}$$

$$\boxed{I_1 = \frac{\varepsilon}{R_1}\left(1 - e^{-R_1/L\, t}\right)}$$