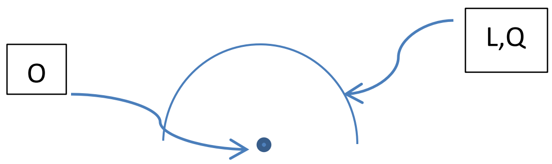
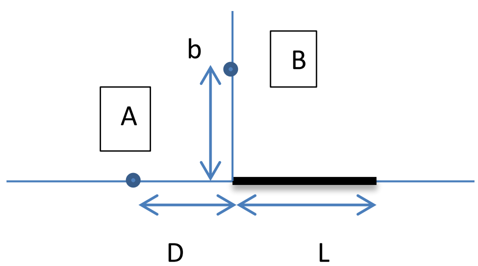
Problem Set 3 --Electric Potential

1. Four point charges each of charge Q are located at the corners of a square of side a. Find
   a. The electric potential (assuming it is zero infinitely far from the square) at the center of the square due to the four charges
   b. The work required to bring a fifth charge q from infinity to the center of the square.
2. Four identical particles, each of charge q and mass m are released from rest at the vertices of a square of side L. How fast is each particle moving when their distance from the center of the square doubles?
3. A uniformly charged insulating rod of length L is bent into a semicircle as shown. The rod has a total charge of Q. Find the electric potential at O, the center of the semicircle. You may assume that the electric potential at infinity is zero.



4. A rod of length L lies along the x axis with its left end at the origin. It has a uniform charge density λ. Calculate:
   a. The electric potential at point $A = (x = -d, y = 0)$.
   b. The electric potential at point $B = (0, b)$ as shown.



5. A long metal cylinder with radius a is supported on an insulating stand on the axis of a long, hollow metal tube with radius b. The positive charge per unit length on the inner cylinder is λ and there is an equal negative charge per unit length $(-\lambda)$ on the outer cylinder.
   a. Calculate the potential V(r) everywhere in space. That is where $r < a$, $a < r < b$, and $r > b$. Take $V = 0$ at $r = b$.
   b. Show that the potential of the inner cylinder with respect to the outer is $V_{ab} = \frac{\lambda}{2\pi\pi}\ln\frac{b}{a}$
   c. Use the result from part a to show that the electric field at any point between the cylinders has magnitude $E(r) = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r}$
   d. What is the potential difference between the two cylinders if the outer cylinder has no net charge?
6. (a.) Show that V for a spherical shell of radius R, that has charge q distributed uniformly over its surface, is the same as V for a solid conductor with radius R and charge q. (b) You rub an inflated balloon on the carpet and it acquires a potential that is 1560 V lower than its potential before it became charged. If the charge is uniformly distributed over the surface of the balloon and if the radius of the balloon is 15 cm, what is the net charge on the balloon?

7. A metal sphere with radius r_a is supported on an insulating stand at the center of a hollow, metal, spherical shell with radius r_b. There is charge $+q$ on the inner sphere and charge $-q$ on the outer spherical shell.
    a. Calculate the electric potential V(r) everywhere in space. That is, for $r < r_a$, $r_a < r < r_b$ and for $r > r_b$. Take $V = 0$ at infinity.
    b. Show that the potential of the inner sphere with respect to the outer is $V_{ab} = \frac{q}{4\pi\epsilon}(\frac{1}{r_a} - \frac{1}{r_b})$
    c. Use the result from part a to show that the electric field at any point between the spheres has magnitude $E\ (r) = \frac{V_{ab}}{(\frac{1}{r_a}-\frac{1}{r_b})}\frac{1}{r^2}$
8. A disk with radius R has uniform surface charge density σ.
    a. By regarding the disk as a series of thin concentric rings, calculate the electric potential V at a point on the disk's axis a distance x from the center of the disk. Assume that the potential is zero at infinity.
    b. Calculate $-\frac{\partial V}{\partial x}$
9. A solid sphere of radius R contains a total charge Q distributed uniformly throughout its volume. Find the energy needed to assemble this charge by bringing infinitesimal charges from far away. (Hint: After you have assembled a charge q in a sphere of radius r, how much energy would it take to add a spherical shell of thickness dr having charge dq? Then integrate the total energy.)