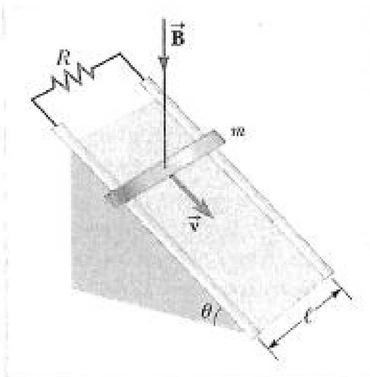
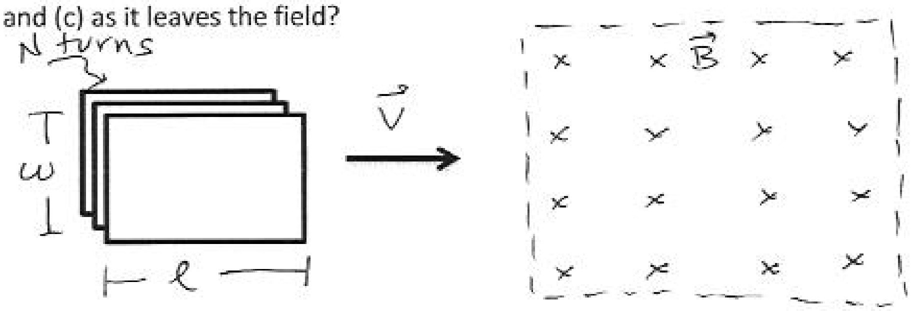
Homework Set 8 – Faraday's Law of Induction

1. A bar of mass *m* slides without friction on a pair of conducting rails separated by a distance *l* and located on an inclined plane that that makes an angle θ with the horizontal as shown in the diagram. The rails are connected to a resistor *R* and a uniform magnetic field ***B*** is directed downward ***(-j-hat)*** over the entire region through which the bar moves. With what constant speed *v* does the bar slide along the rails?

$$v = \frac{mgR \tan\theta}{B^2 l^2 \cos\theta}$$



2. A rectangular coil with resistance *R* has *N* turns, each of length *l* and width *w* as shown. The coil moves into a uniform magnetic field ***B*** with constant velocity ***v***. What are the magnitude and direction of the total magnetic force on the coil (a) as it enters the magnetic field, (b) as it moves within the field and (c) as it leaves the field?



(a) $\vec{F}$ (opposite velocity) $= \frac{NB^2 w^2 v}{R}$, ←

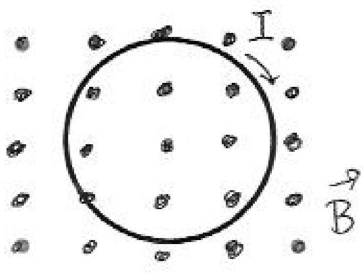
(b) 0

(c) $\vec{F} = \frac{NB^2 w^2 v}{R}$, →

3. A circular loop of wire of resistance *R* and radius *a* is in a uniform magnetic field directed out of the pages as shown. If a clockwise current of I is induced in the loop, (a) is the magnetic field increasing or decreasing in time? (b) find the rate at which the field is changing with time.

(a) increasing

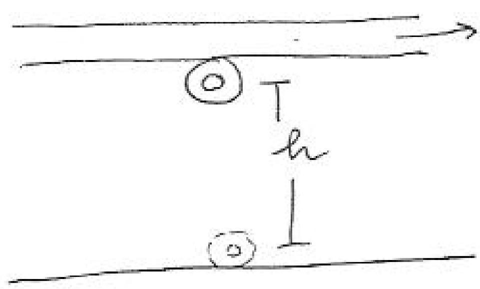
(b) $\frac{dB}{dt} = \frac{IR}{\pi a^2}$



4. A rectangular loop of area A is placed in a region where the magnetic field is perpendicular to the plane of the loop. The magnitude of the field is allowed to vary in time according to $B = B_o e^{-t/\tau}$ where B_o and τ are constants. The field has the constant value B_o for t < 0. Find the emf induced in the loop as a function of time.

$$\mathcal{E} = \frac{AB_o}{\tau} e^{-t/\tau}$$

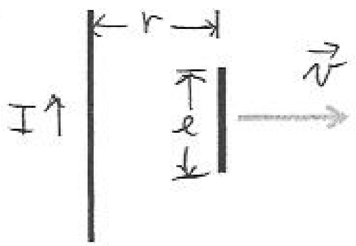
5. A small circular washer of radius a is held directly below a long, straight wire carrying current I. The washer is located at height h above a table. Assume the magnetic field is nearly constant of the area of the washer and equal to the magnetic field at the center of the washer. The washer is dropped from rest. Find the magnitude of the induced emf over the time interval between its release and the moment it hits the tabletop. What is the direction of the induced current in the washer?



$$|\varepsilon| = \frac{\mu_0 I a^2 g t}{2(a + \frac{1}{2}gt^2)^2}$$

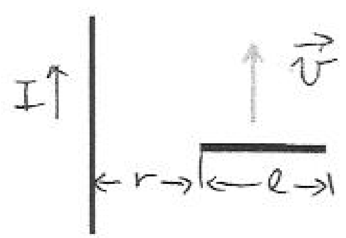
clockwise

6. A conducting rod moves with a constant velocity in a direction perpendicular to a long, straight wire carrying current I as shown in the figure. Show that the magnitude of the emf generated between the ends of the rod is $|\varepsilon| = \frac{\mu_0 v I l}{2\pi r}$



7. A conducting rod of length l moves with velocity V parallel to a long wire carrying a steady current I. The axis of the rod is maintained perpendicular to the wire with the near end a distance r away. Show that the magnitude of the emf induced in the rod is

$$|\varepsilon| = \frac{\mu_0 I v}{2\pi} \ln(1 + \frac{l}{r})$$



8. An ac generator's rectangular loop of dimensions *a* and *b* has *N* turns. The loop is connected to slip rings and rotates with an angular velocity ω in a uniform magnetic field ***B***. (a) Show that the potential difference between the two slip rings is $\varepsilon = NBab\omega \sin \omega t$. (b) If *a* = 1.0 cm, *b* = 2.0 cm, *N* = 1000, *B* = 2T, at what angular frequency ω must the coil rotate to generate an emf whose maximum value is 110V? (ans: 275 rads/s)

(b) 275 rad/s