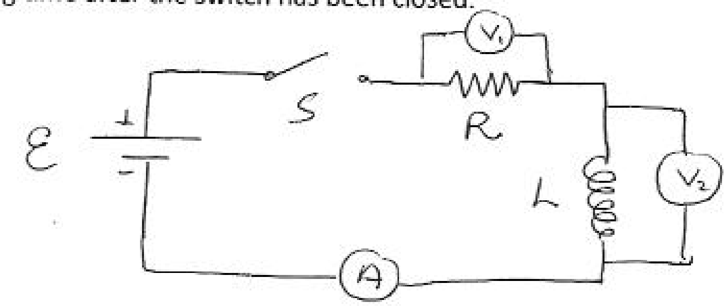
Problem Set 9 – nductance

1. A small solid conductor with radius a is supported by insulating, nonmagnetic disks on the axis of a thin walled tube with inner radius b. The inner and outer conductors carry equal currents i in opposite directions. (a) Using with Ampere's Law, find an expression for the magnetic field in the region between the conductors. (b) Write and expression for the flux $d\varphi$ through a narrow strip of length l parallel to the axis, of width dr at a distance r from the axis and lying in the plane containing the axis. (c) integrate to find the total flux produced by a current i in the central conductor. (d) find the inductance. $\left(L = \frac{\mu_0 l}{2\pi} \ln \frac{b}{a}\right)$

2. In the circuit shown, find the readings in the ammeter and voltmeters the instant switch S is closed and then a long time after the switch has been closed.



(a) the instant S is closed

$A = 0$

$V_1 = 0$

$V_2 = \varepsilon$

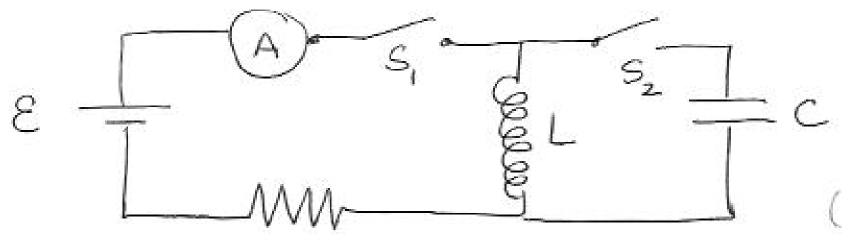
(b) S is closed for a long t.

$I = V/R = \varepsilon/R$

$V_1 = \varepsilon$

$V_2 = 0$

3. In the circuit shown below, the switch S_1 has been closed for a long enough time so that the current reads a steady value of I_o. Suddenly, switch S_1 is opened and S_2 is closed at the same instant. (a) what is the maximum charge that the capacitor will receive? (b) what is the current in the inductor at this time?

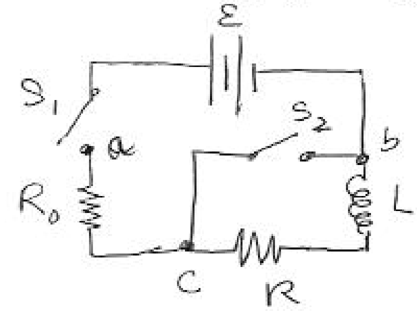


$I_0 = \frac{\varepsilon}{R}$

$\frac{1}{2} L I_0^2 = \frac{1}{2} \frac{q_{max}^2}{C}$

(b) $I \Rightarrow 0$

4. Consider the circuit shown. S_1 has been closed for a long time with switch S_2 open. Just after switch S_2 is closed, what are the values of the potential V_{ac} and V_{cb}? And what are the currents through R_o, R and S_2? (b) a long time after S_2 is closed, what are V_{ac} and V_{cb} and the currents through R_o, R and S_2? Derive an expression for the currents through R_o, R and S_2 as functions of time.



(a) $V_{ac} = \varepsilon$; $V_{bc} = 0$

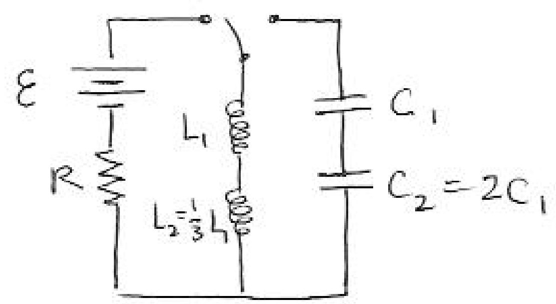
(b) $V_{ac} = \varepsilon$; $V_{bc} = 0$

$i_R = \frac{\varepsilon}{R_0 + R} e^{-R/L\, t}$

$i_{S_2} = \frac{\varepsilon}{R_0} - \frac{\varepsilon}{R_0 + R} e^{-R/L\, t}$

$i_{R_0} = \frac{\varepsilon}{R_0}$

5. In the circuit shown below, neither the battery nor the inductors have appreciable resistance, the capacitors are initially uncharged and the switch has been in position 1 for a long time. (a) what is the current in the circuit? (b) the switch is suddenly flipped to position 2, find the maximum charge that each capacitor will receive.



(a) $I = \frac{\varepsilon}{R}$

(b) $q_{max} = \frac{2}{3} I_0 \sqrt{2L_1C_1}$

6. At some instant in an oscillating *LC* circuit, three-fourths of the total energy is stored in the magnetic field of the inductor. (a) in terms of the maximum charge on the capacitor, what is the charge on the capacitor at this instant? (b) in terms of the maximum current in the inductor, what is the current in the inductor at this instant? $Q = \frac{1}{2} Q_{max}$; $I = \frac{\sqrt{3}}{2} I_{max}$

7. Starting with the magnetic field energy $U_m = \frac{1}{2} L I^2$, find the magnetic energy density for a toroidal solenoid filled with magnetic material.

8. Consider the coaxial cable of problem 1. (a) Starting with the magnetic energy density, find the energy stored in a thin, cylindrical shell between the two conductors. Let the cylindrical shell have inner radius *r*, outer radius *r + dr* and length *l*. (b) integrate your result over the volume between the two conductors to find the total energy stored in the magnetic field for a length *l* of the cable. (c) Use your result to calculate the inductance *L* and compare to problem 1.

→ NOT TERRIFICALLY SATISFYING!

IN ORDER TO GET u_m in the classic form $u_B = \frac{1}{2}\frac{B^2}{\mu}$ you must assume the magnetic field in the interior of the solenoid is uniform!

Thus $|\vec{B}| = \frac{\mu N I_0}{2\pi r}$ and $\Phi = \int \vec{B} \cdot \hat{n}\, dA = BA$

and $V_d = A \cdot 2\pi r$